Lecture 4: Logistic Regression

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USC CSCI 544 Applied NLP

Sep 5, Fall 2024

Some slides adapted from Dan Jurafsky and Chris Manning and Xuezhe Ma

Lecture Outline

- Recap
  - Smoothing
  - Basics of Supervised Machine Learning
    - Data: Preprocessing and Feature Extraction
- Quiz
- Announcements
- Basics of Supervised Machine Learning
  I. Data: Preprocessing and Feature Extraction
  II. Model:
      I. Logistic Regression
  III. Loss
  IV. Optimization Algorithm
  V. Inference

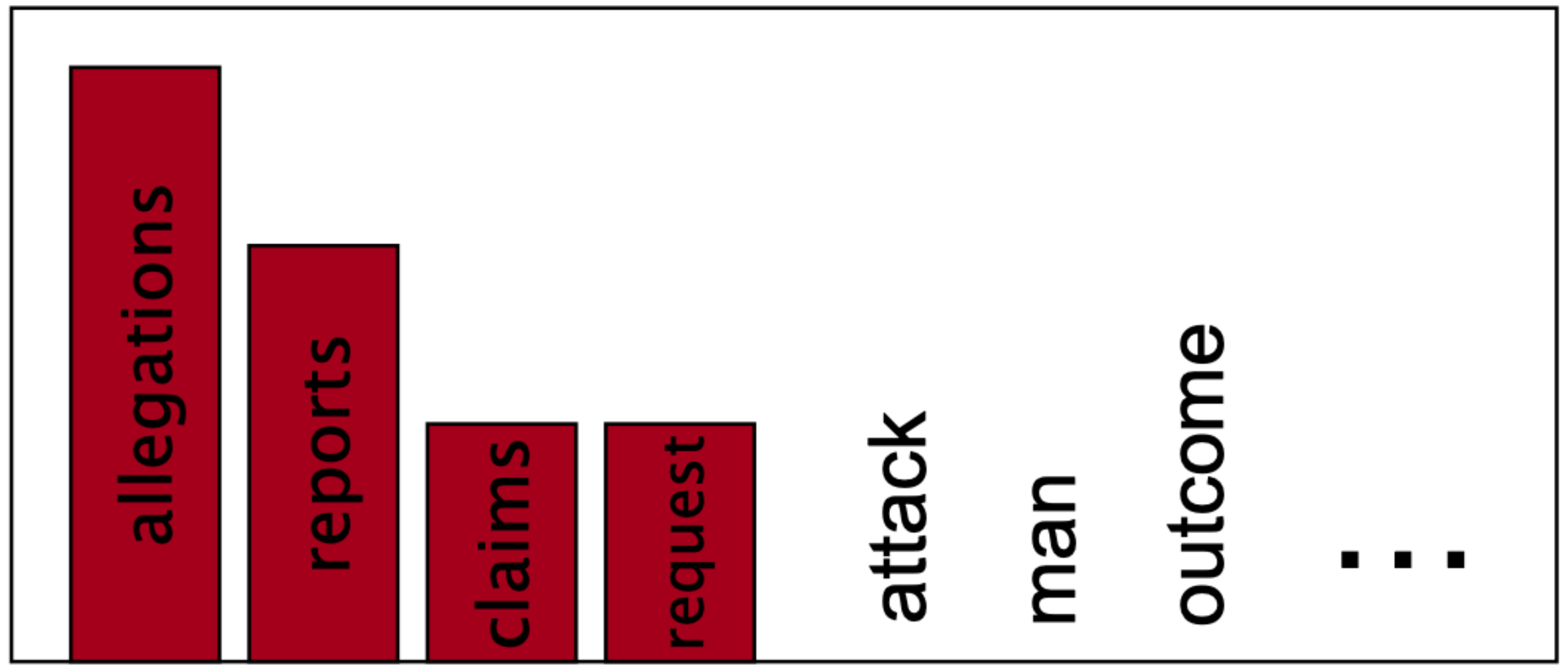
Recap: Smoothing

USC Viterbi

Smoothing ~ Massaging Probability Masses

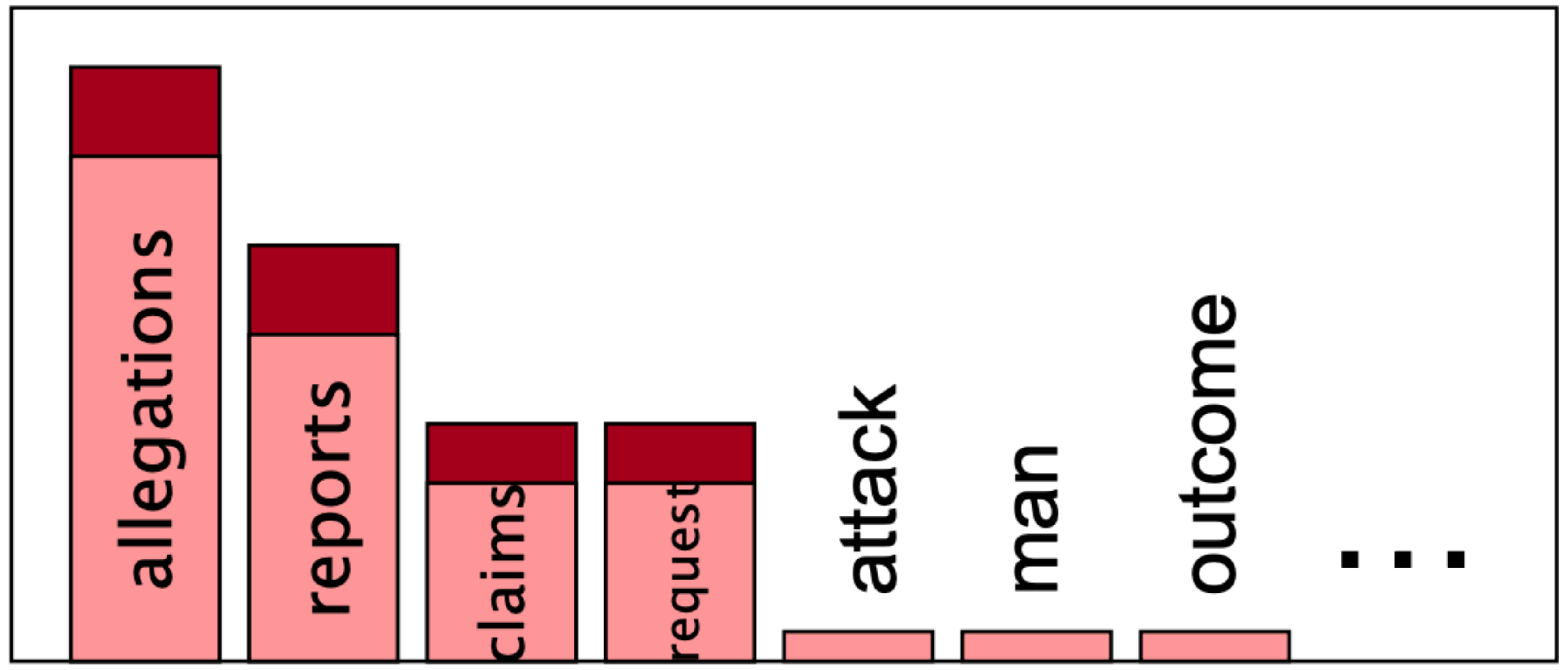
When we have sparse statistics: $Count(w \mid \text{denied the})$

3 allegations
2 reports
1 claims
1 request
7 total



Steal probability mass to generalize better: $Count(w \mid \text{denied the})$

2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 other
7 total



Add-One Estimation

Laplace smoothing

1. Pretend we saw each n-gram one more time than we did
2. Just add one to all the n-gram counts!
3. All the counts that used to be zero will now have a count of 1…

Add-1 estimate for Unigrams

$$P_{Add-1}(w_i) = \frac{c(w_i) + 1}{\sum_w (c(w) + 1)} = \frac{c(w_i) + 1}{V + \sum_w c(w)}$$

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Add-1 estimate for Bigrams

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1} w_i) + 1}{c(w_{i-1}) + V}$$

Original vs Add-1 smoothed bigram counts

Original, Raw

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Reconstructed

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Big change to the counts!

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Big change to the counts!

Perhaps 1 is too much, add a fraction?

Add-k smoothing

k is a hyperparameter

Linear Interpolation

Simple Interpolation

$$\hat{P}(w_i | w_{i-2}w_{i-1}) = \lambda_1 P(w_i) + \lambda_2 P(w_i | w_{i-1}) + \lambda_3 P(w_i | w_{i-2}w_{i-1})$$

$$\sum_k \boxed{\lambda_k} = 1$$

Hyperparameters!

Context-Conditional Interpolation

$$\hat{P}(w_i | w_{i-2}w_{i-1}) = \lambda_3(w_{i-2}^{i-1}) P(w_i | w_{i-2}w_{i-1}) + \lambda_2(w_{i-2}^{i-1}) P(w_i | w_{i-1}) + \lambda_1(w_{i-2}^{i-1}) P(w_i)$$

Different hyperparameters for different bigrams (context conditional)!

Recap: Basics of Supervised Machine Learning

Ingredients of Supervised Machine Learning

I. **Data** as pairs $(x^{(i)}, y^{(i)})$ s.t $i \in \{1...N\}$
   - $x^{(i)}$ usually represented by a feature vector $\mathbf{x}^{(i)} = [x_1, x_2, ..., x_d]$,
     - e.g. word embeddings

II. **Model**
   - A classification function that computes $\hat{y}$, the estimated class, via $p(y \mid x)$
     - e.g. logistic regression, naïve Bayes

III. **Loss**
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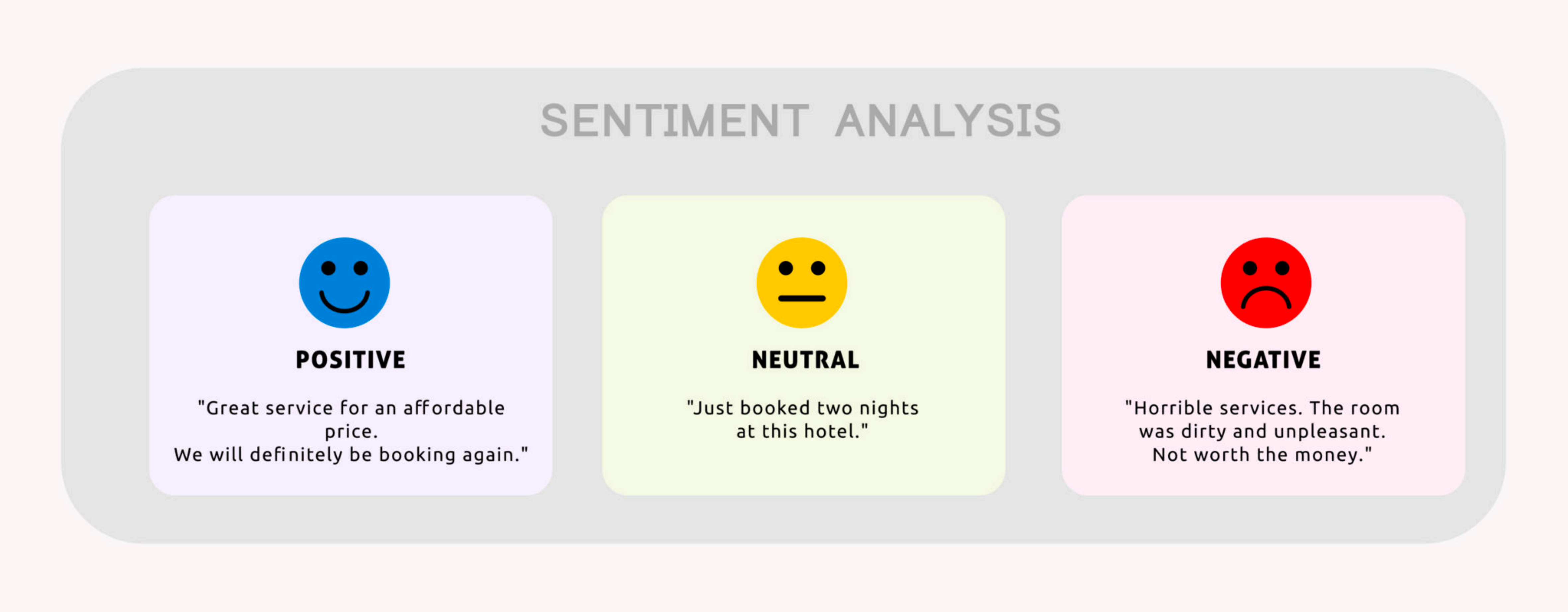
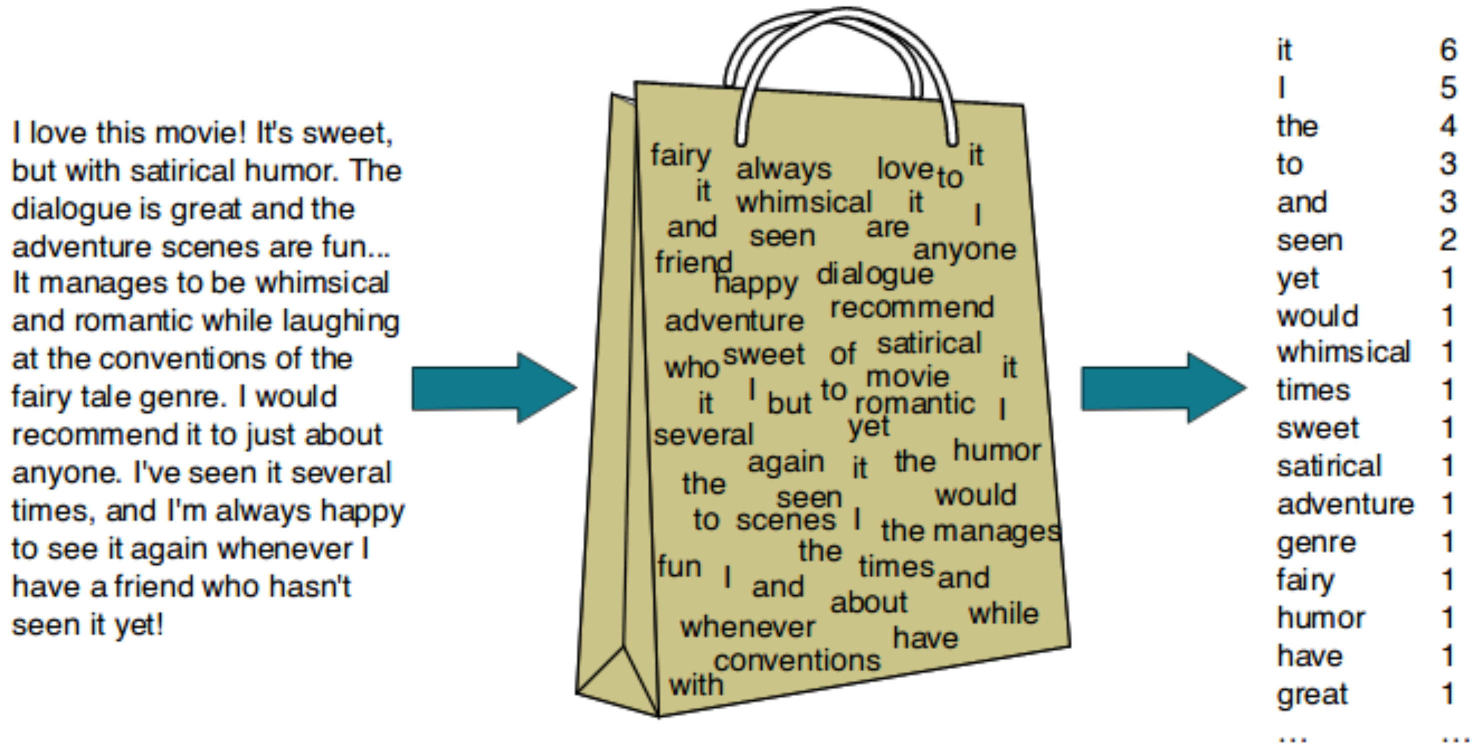
Learning Phase

Features in Classification



- Examples of feature x_i
  - x_i ="review contains 'awesome'"; $w_i = +10$
  - x_j ="review contains 'abysmal'"; $w_j = -10$
  - x_k ="review contains 'mediocre'"; $w_k = -2$
- Each x_i is associated with a weight w_i which determines how important x_i is
  - (For predicting the positive class)
- May be
  - manually configured or
  - automatically inferred, as in modern architectures

Another type of feature representation: Bag of Words



- With a word vocabulary of k words, BoW represents each doc (e.g., review) into a vector of integers
  - You may choose which k words, depending on the application
- $\mathbf{x} = [x_1, \ldots, x_k], \quad x_i \in 0,1,2,\ldots$
  - $x_i = j$ indicates that word i appears j times in the doc (e.g., review)

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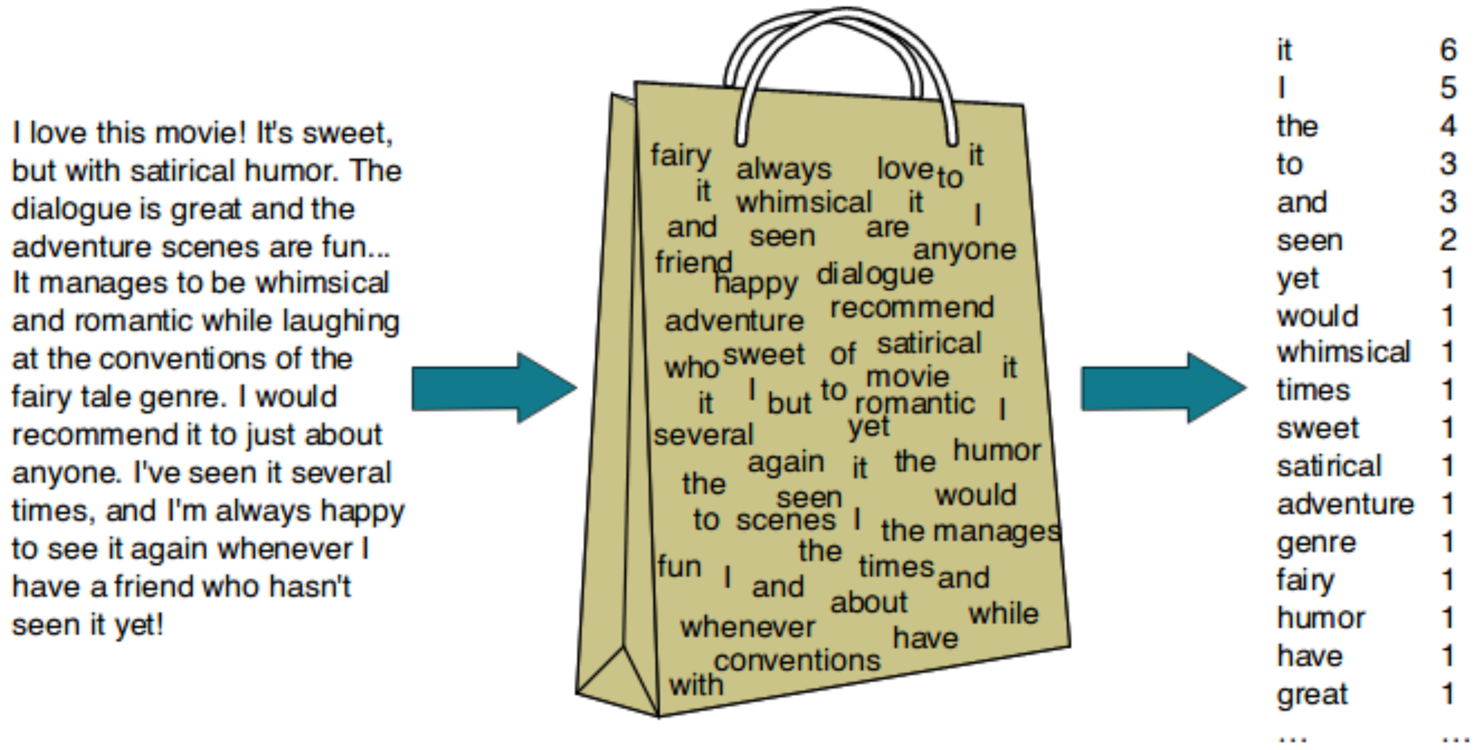

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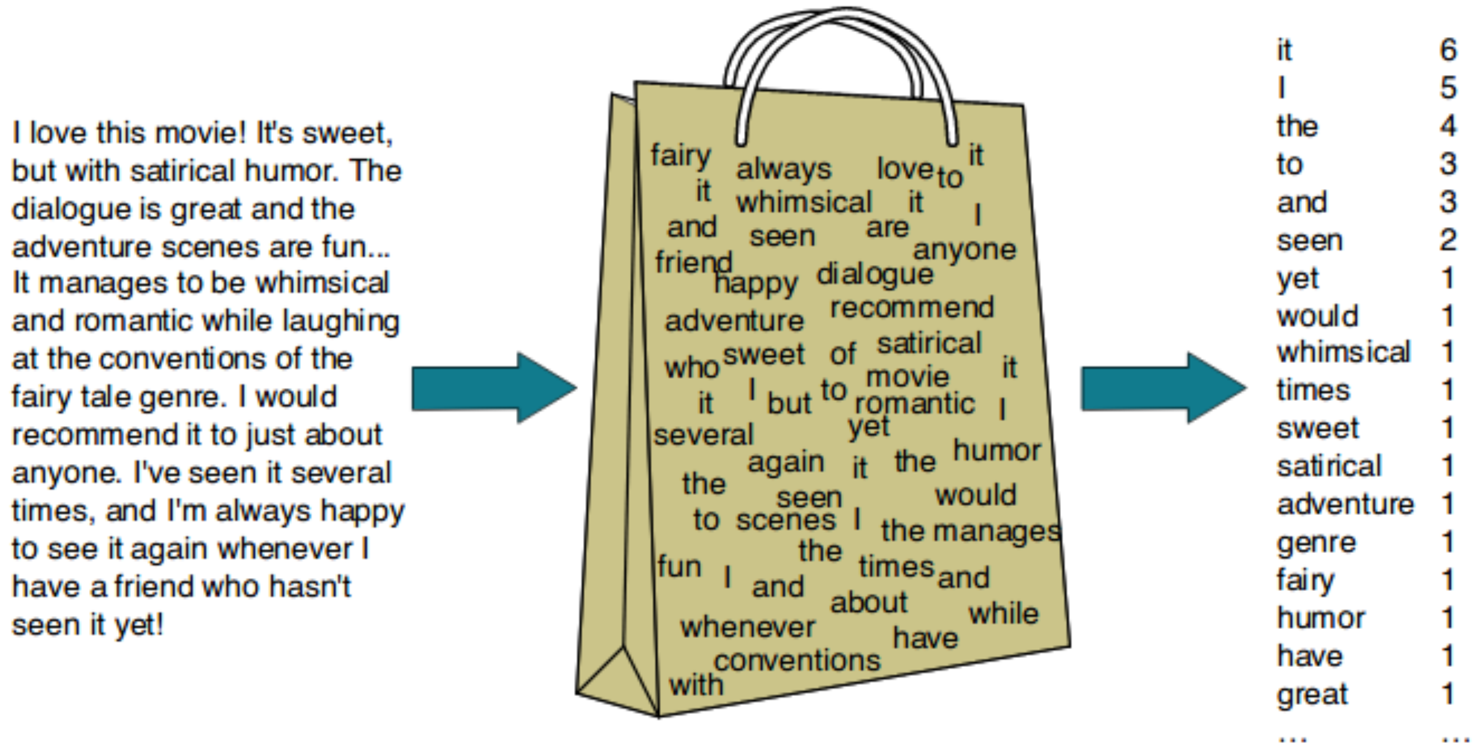

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Feature Definition = [good, bad, nice, ugly, love, hate, complements, coarse, itchy]

"I love this shirt because it is nice and warm. The fabric is also nice and the color complements my skin tone."

$$\mathbf{x} = [0, \quad 0, \quad 2, \quad 0, \quad 1, \quad 0, \quad 1, \quad 0, \quad 0]$$

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- Pros:
  - Simple!
  - Leads to acceptable performance in quite a few settings

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Quiz 1 on Brightspace

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- Brightspace - Subscribe to Discussions etc. if you would like to receive notifications

II. Model: Logistic Regression

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But how to determine the threshold?

We need probabilistic models!

$$P(y = 1 \mid \mathbf{x}; \theta)$$

$$P(y = 0 \mid \mathbf{x}; \theta)$$

$$z = \mathbf{w} \cdot \mathbf{x} + b \qquad z \in \mathbb{R}$$

Solution: Squish it into the 0-1 range

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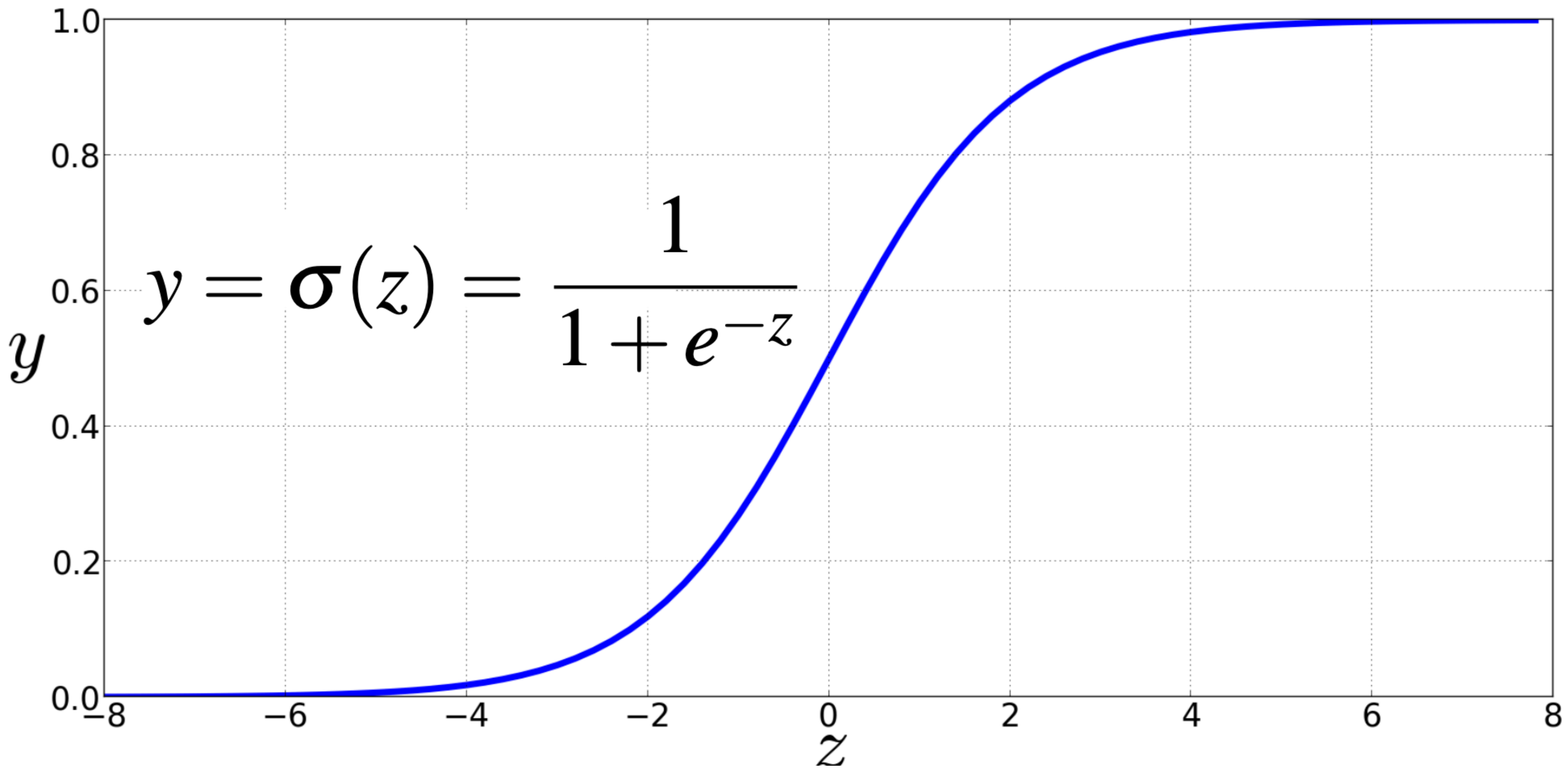
- Sigmoid Function, $\sigma(\cdot)$



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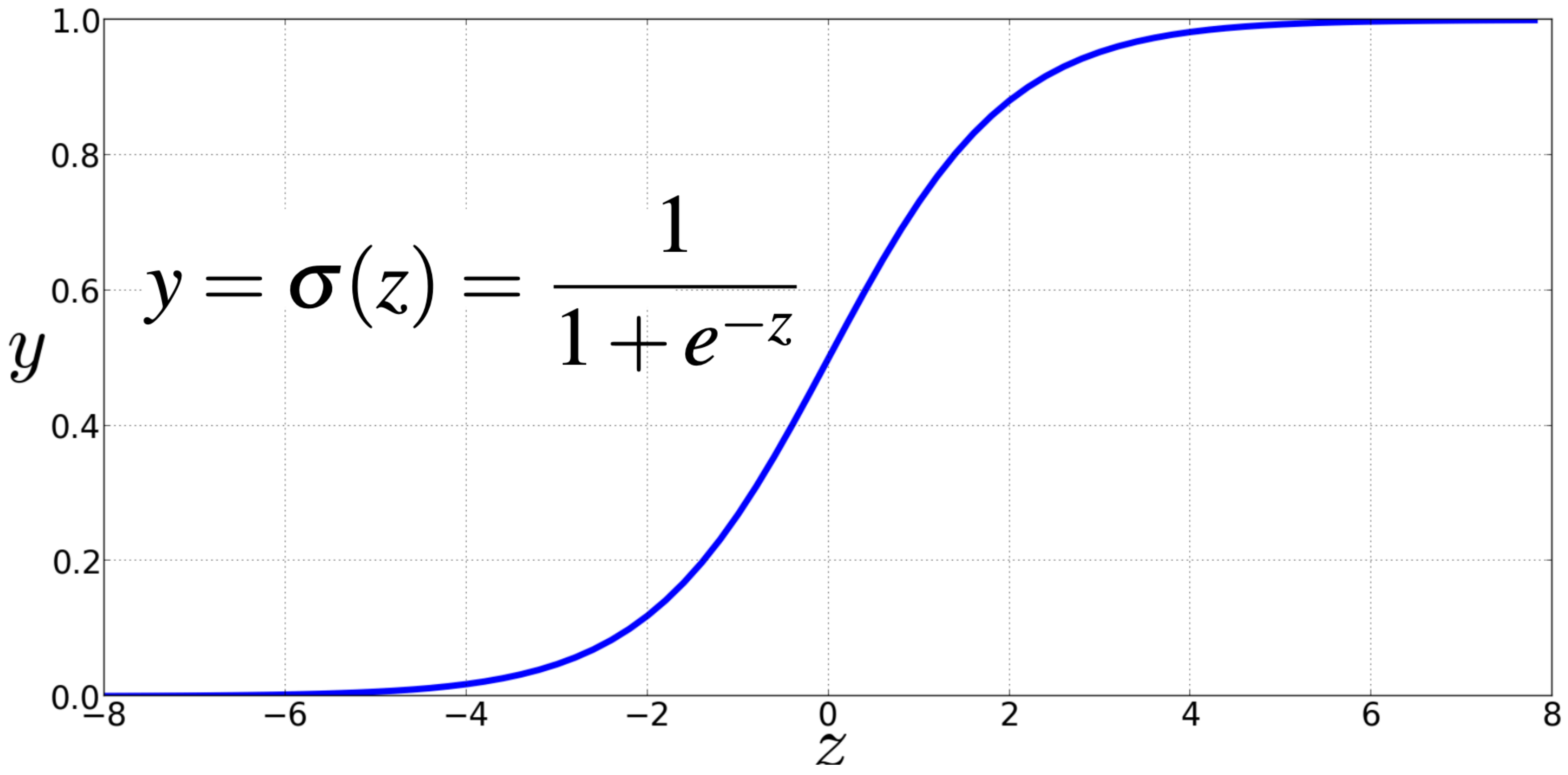
  - Non-linear!



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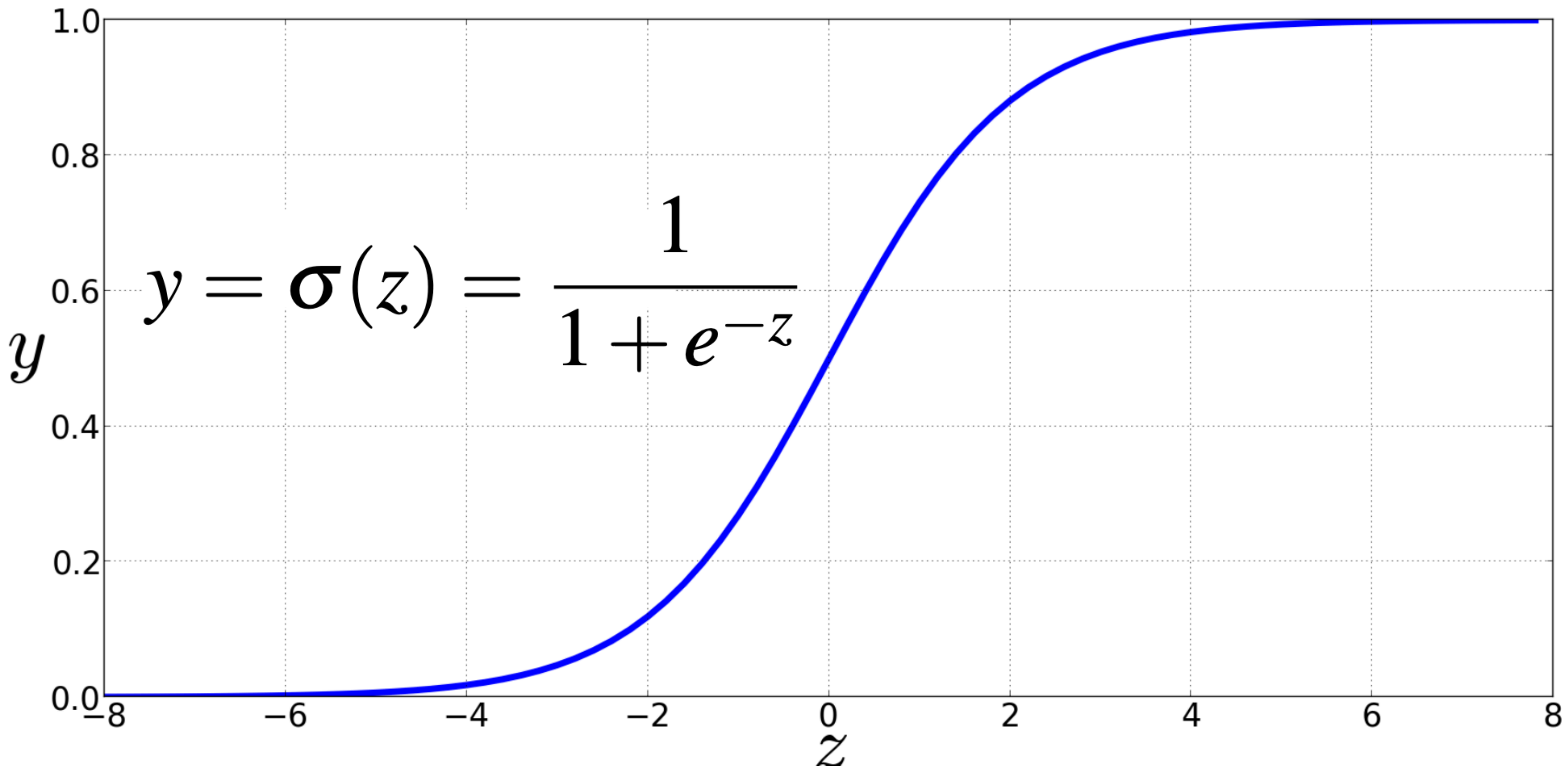

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- Compute z and then pass it through the sigmoid function
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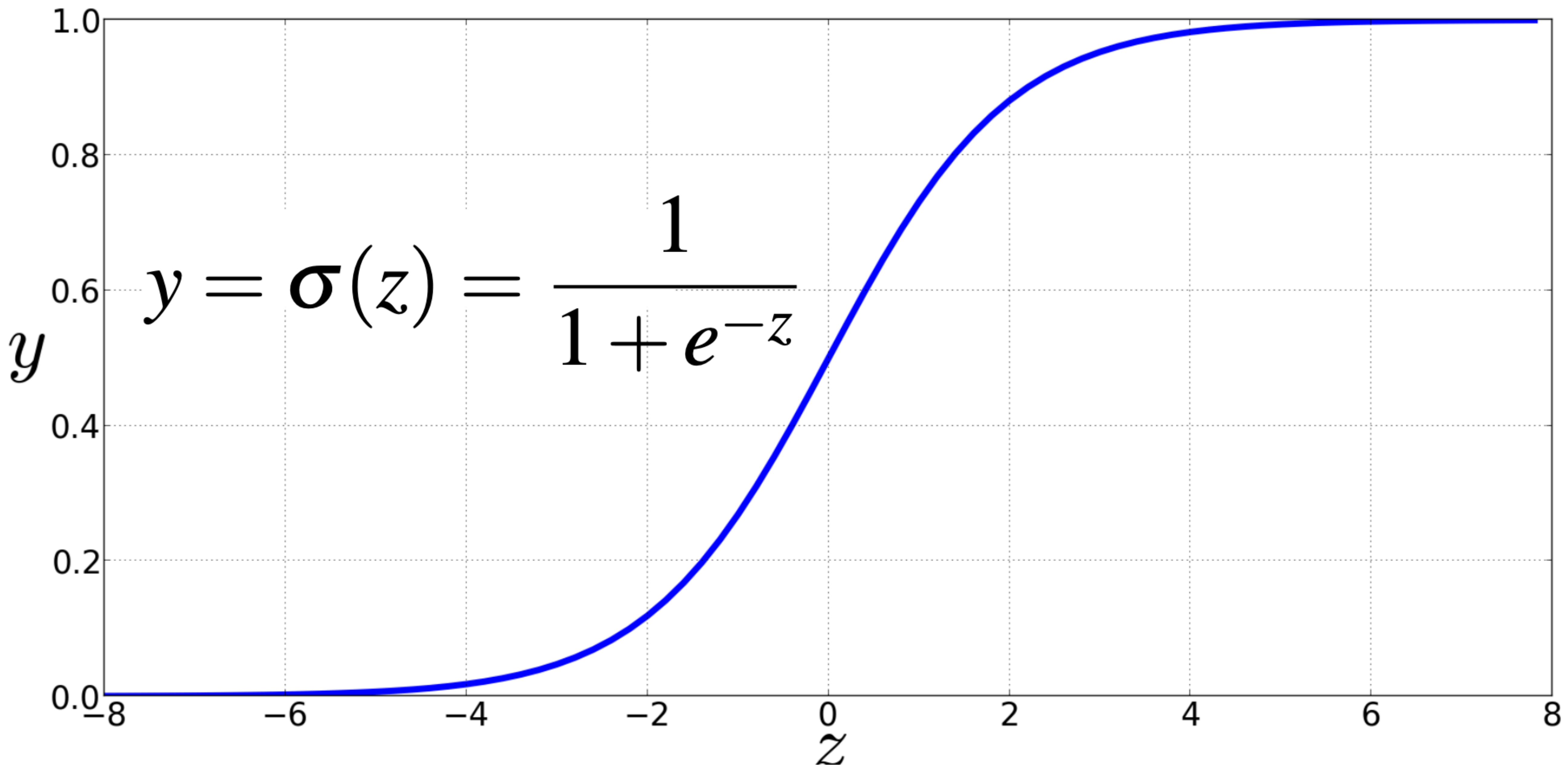

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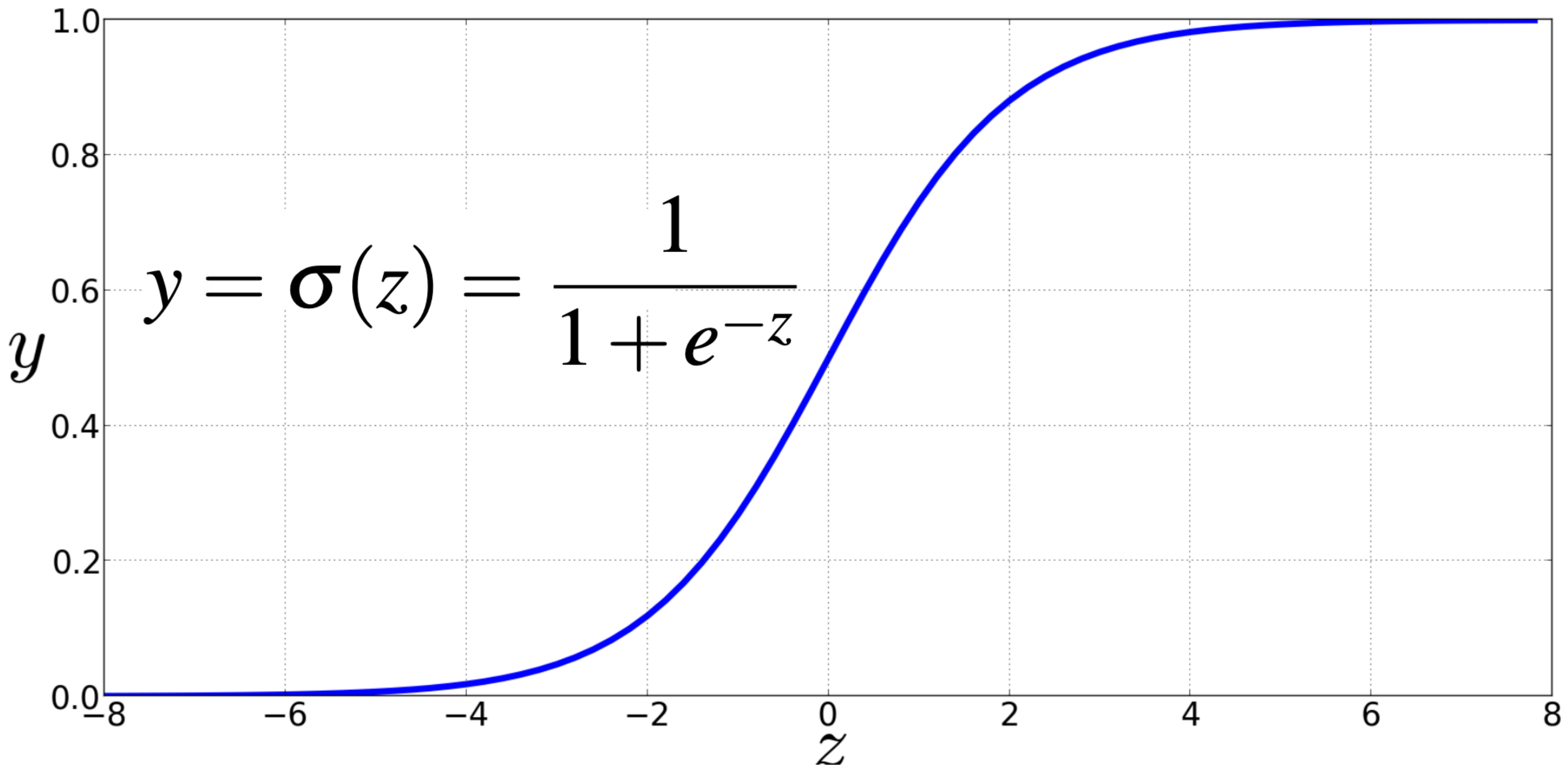
- Sigmoid Function, $\sigma(\cdot)$
  - Non-linear!
- Compute z and then pass it through the sigmoid function
- Treat it as a probability!
- Also, a differentiable function, which makes it a good candidate for optimization (more on this later!)



Sigmoids and Probabilities

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$$P(y = 1 \,|\, \mathbf{x}; \theta) = \sigma(\mathbf{w} \cdot \mathbf{x} + b)$$

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$$\begin{aligned}P(y=0\,|\,\mathbf{x};\theta) &= 1-\sigma(\mathbf{w}\cdot\mathbf{x}+b)\\ &= 1-\frac{1}{1+\exp(-(\mathbf{w}\cdot\mathbf{x}+b))}\\ &= \frac{\exp(-(\mathbf{w}\cdot\mathbf{x}+b))}{1+\exp(-(\mathbf{w}\cdot\mathbf{x}+b))}\\ &= \frac{1}{1+\exp(\mathbf{w}\cdot\mathbf{x}+b)}\\ &= \sigma(-(\mathbf{w}\cdot\mathbf{x}+b))\end{aligned}$$

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Classification Decision

$P(y = 1 \mid \mathbf{x}; \mathbf{w}, b)$

$\mathbf{w} \cdot \mathbf{x} + b$

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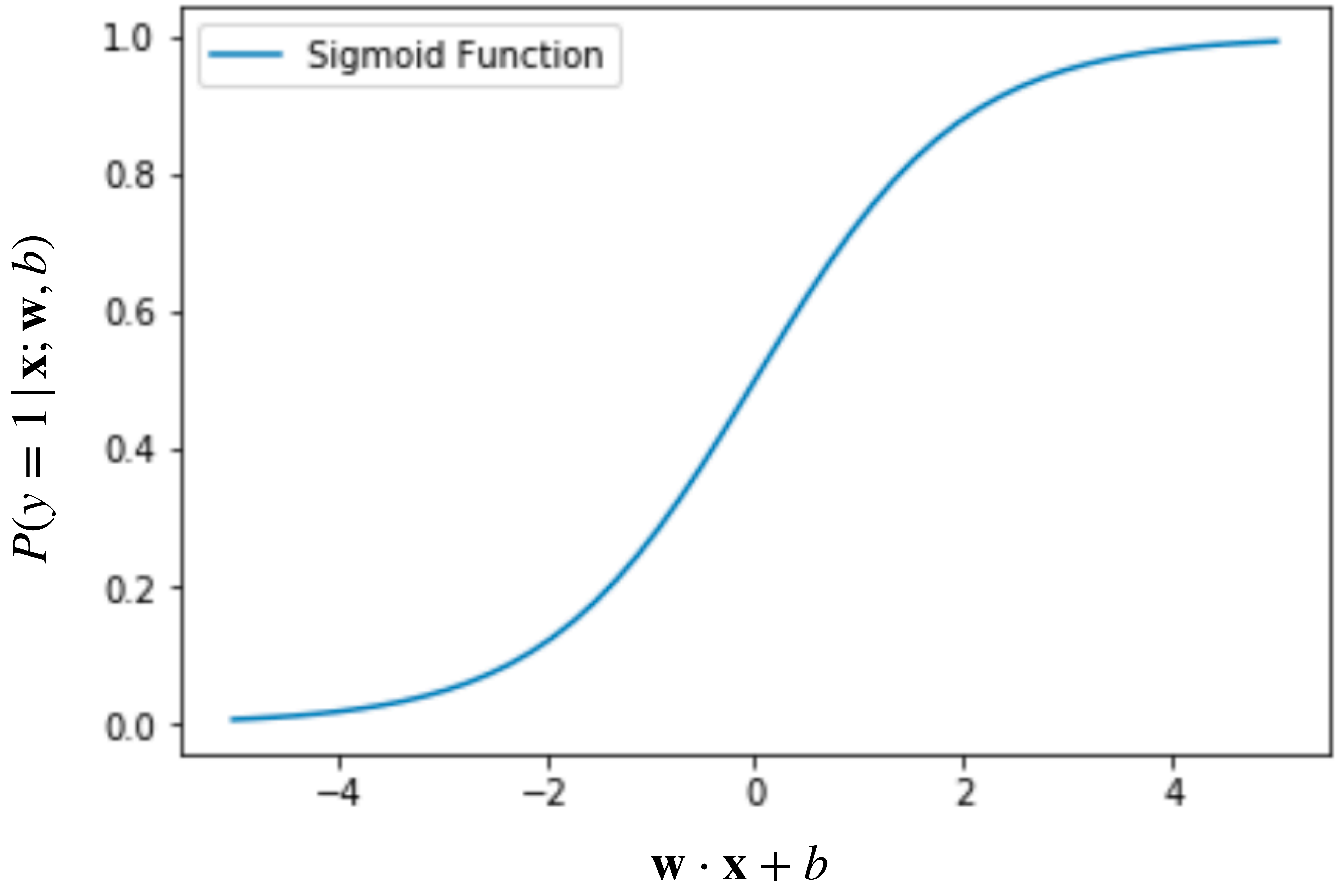

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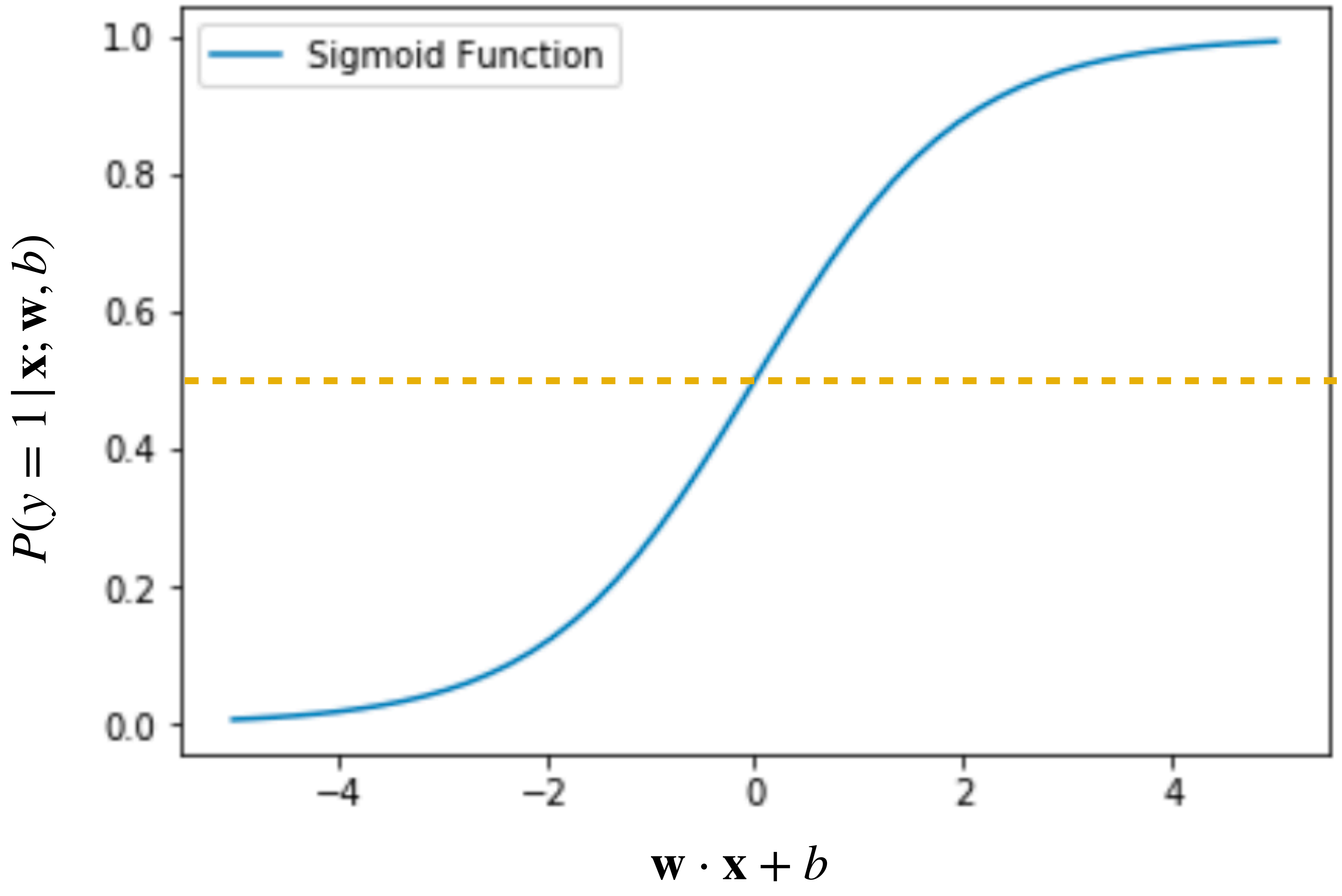

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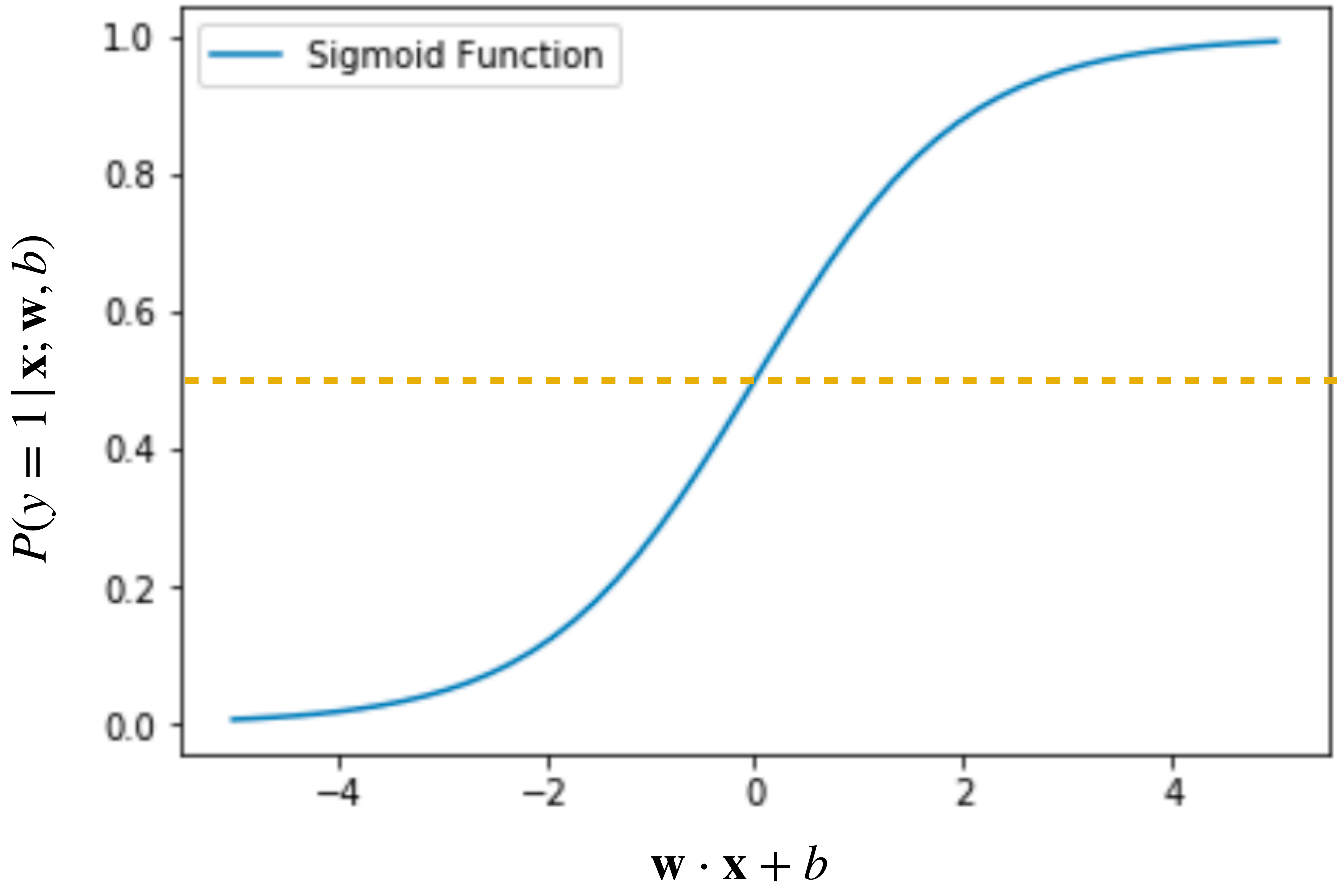

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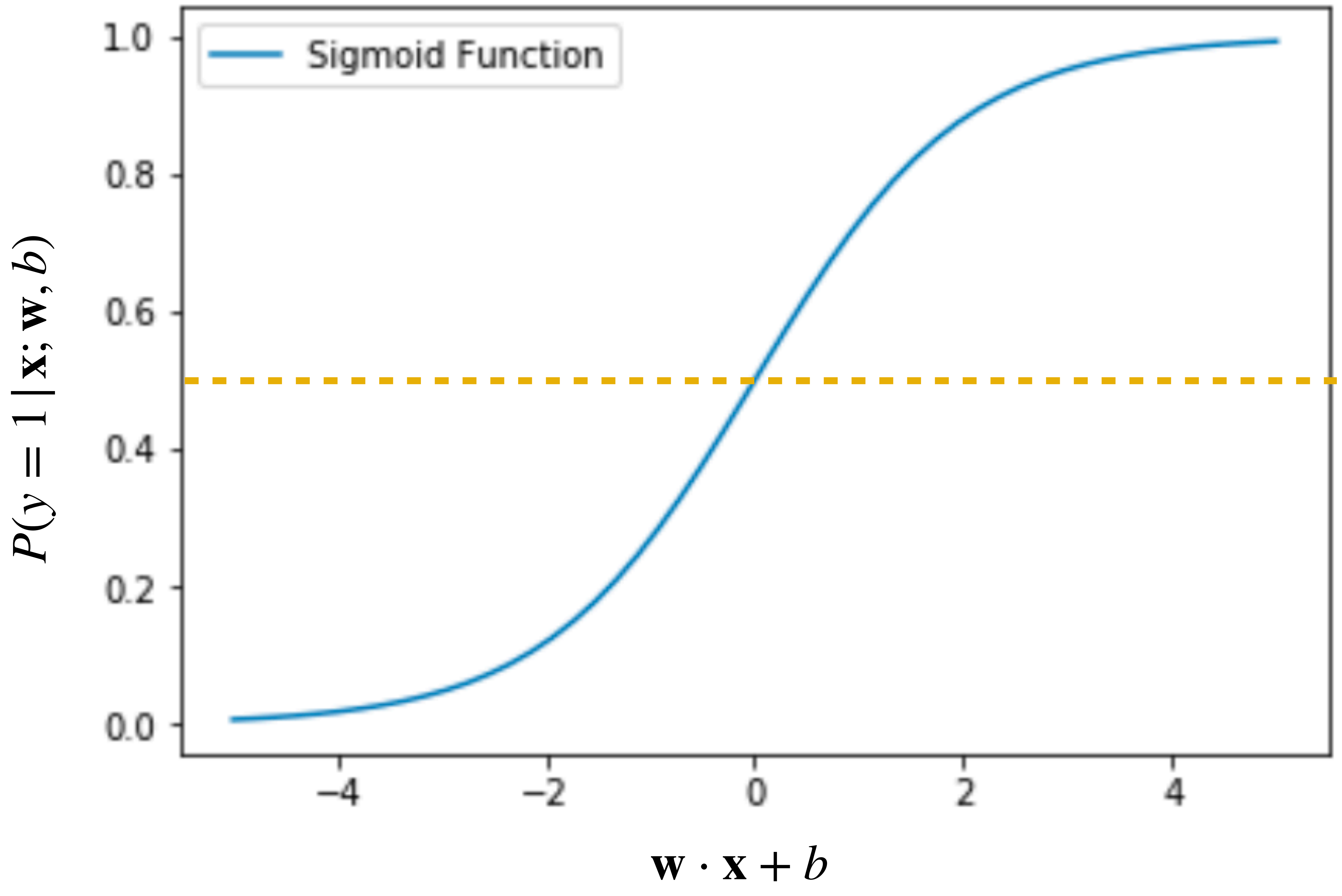

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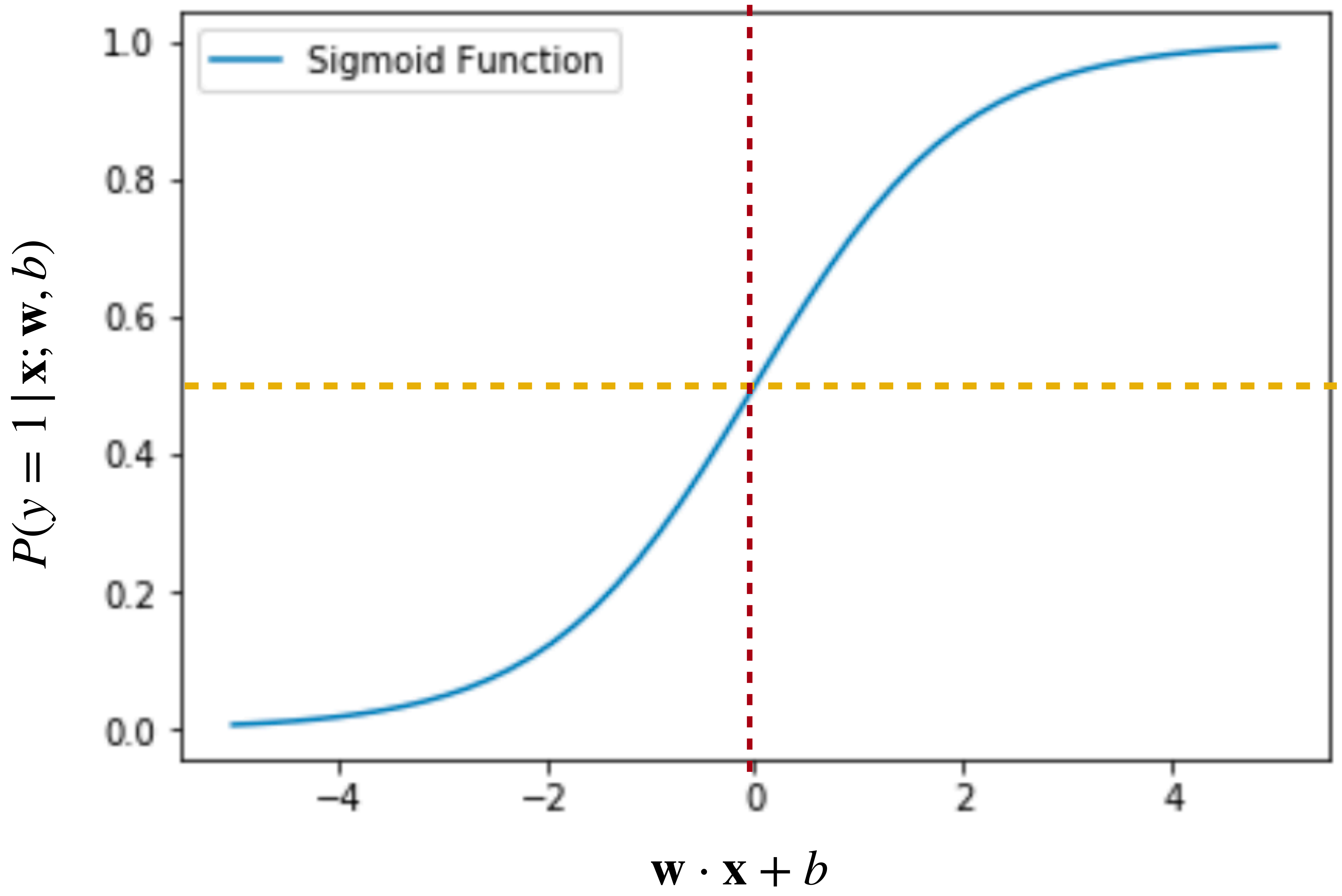

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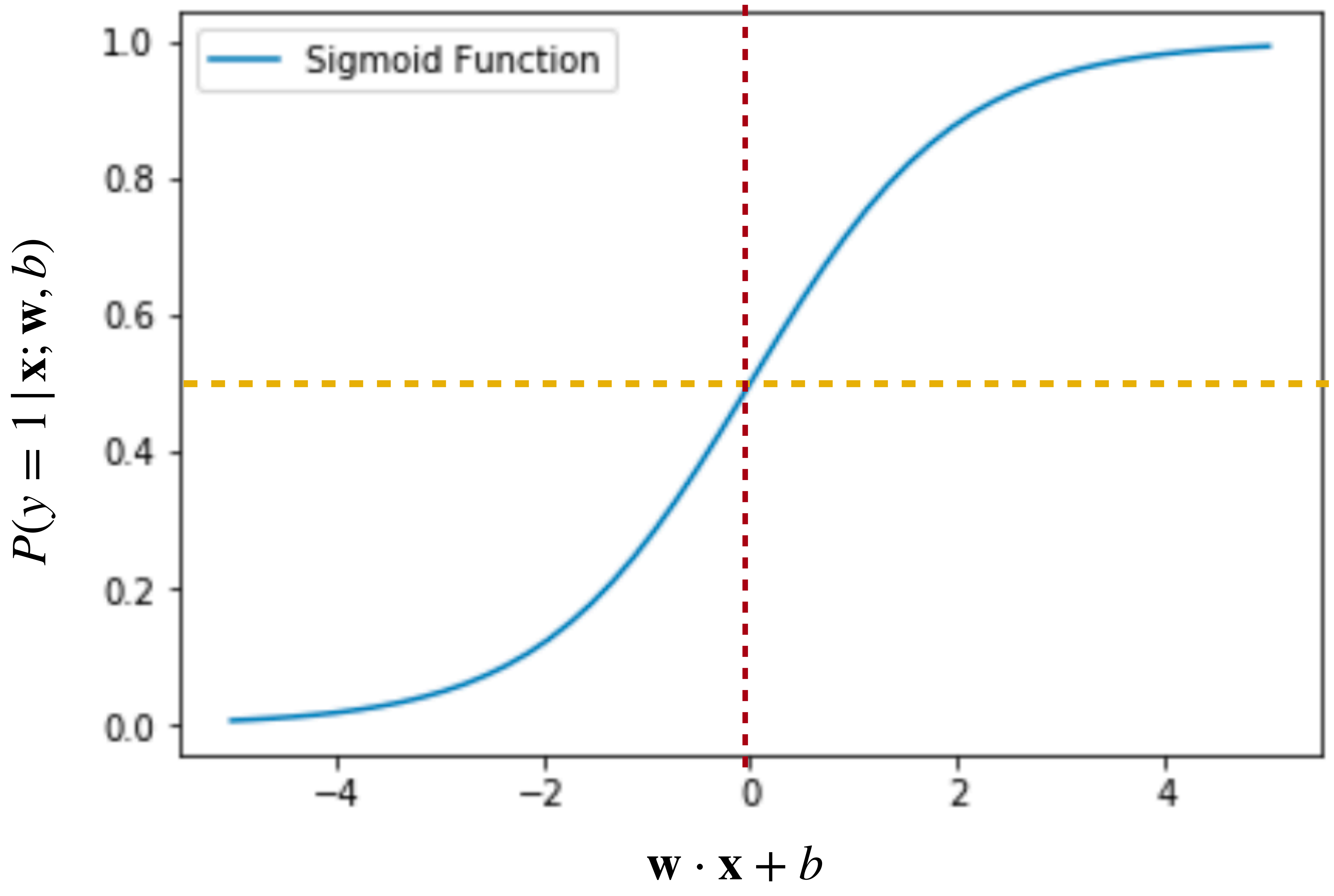
Decision Boundary

$$\hat{y} = \begin{cases} 1 & \textbf{if } \mathbf{w} \cdot \mathbf{x} + b > 0 \\ 0 & \textbf{if } \mathbf{w} \cdot \mathbf{x} + b \leq 0 \end{cases}$$



Another notation

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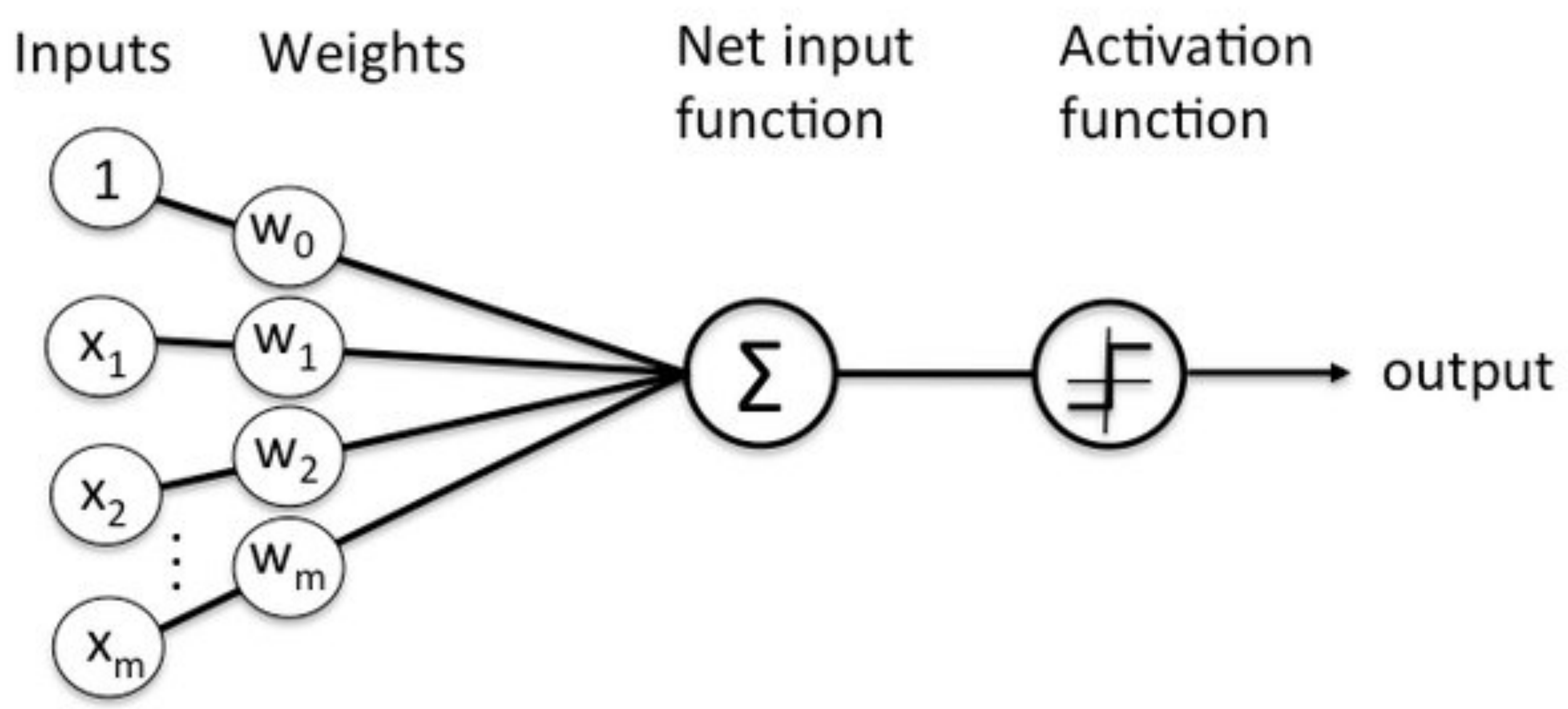

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Loss function

Optimization Algorithm

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III. Loss: Cross-Entropy

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- This difference is called the loss or cost
  - $L(\hat{y}, y)$ = how much $\hat{y}$ differs from y
  - In other words, how much would you lose if you mispredicted
  - Or how much would it cost you to mispredict

Remember maximum likelihood?

Suppose we flip the coin four times and see (H, H, H, T). What is p?

$p = 3/4 = 0.75$ maximizes the probability of data sequence (H,H,H,T)

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Goal: maximize probability of the correct label $p(y \mid x)$

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Why does this work?

Minimizing negative log likelihood

$$
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Cross-Entropy Loss

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Cross-Entropy Loss

Minimizing negative log likelihood

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Measures how well the training data matches the proposed model distribution and how good the model distribution is

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IV. Optimization:
Stochastic Gradient Descent

Our goal: minimize the loss

- Loss function is parameterized by weights: $\theta = [\mathbf{w}; b]$
- We will represent $\hat{y}$ as $f(x; \theta)$ to make the dependence on θ more obvious

$$L_{CE}(f(\mathbf{x}^{(i)}; \theta), y^{(i)})$$

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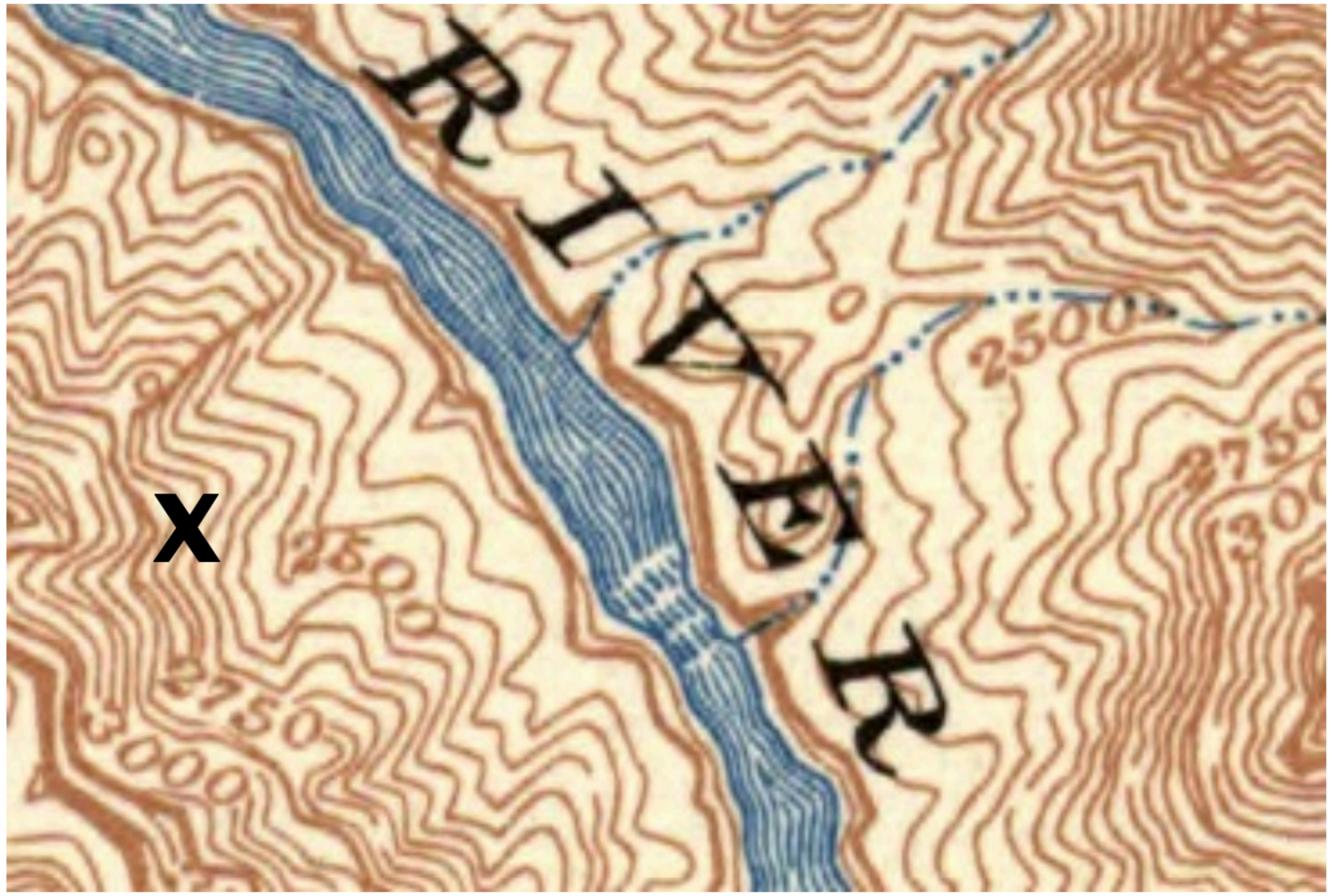
We want the weights that minimize the loss, averaged over all examples:

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{m} \sum_{i=1}^{m} L_{CE}(f(\mathbf{x}^{(i)};\theta), y^{(i)})$$

Intuition for gradient descent

How to get to the bottom of the river canyon?



Intuition for gradient descent

How to get to the bottom of the river canyon?

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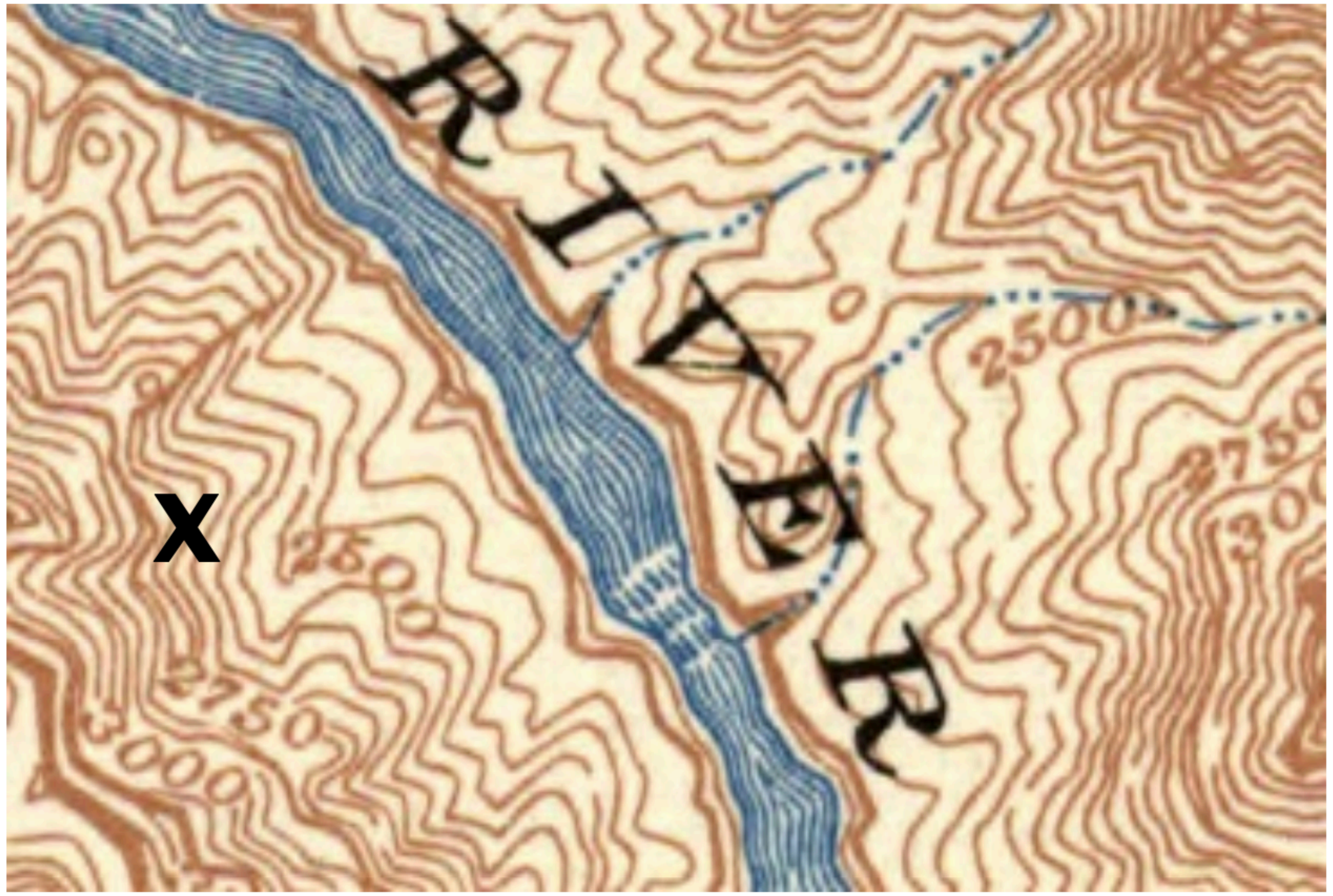

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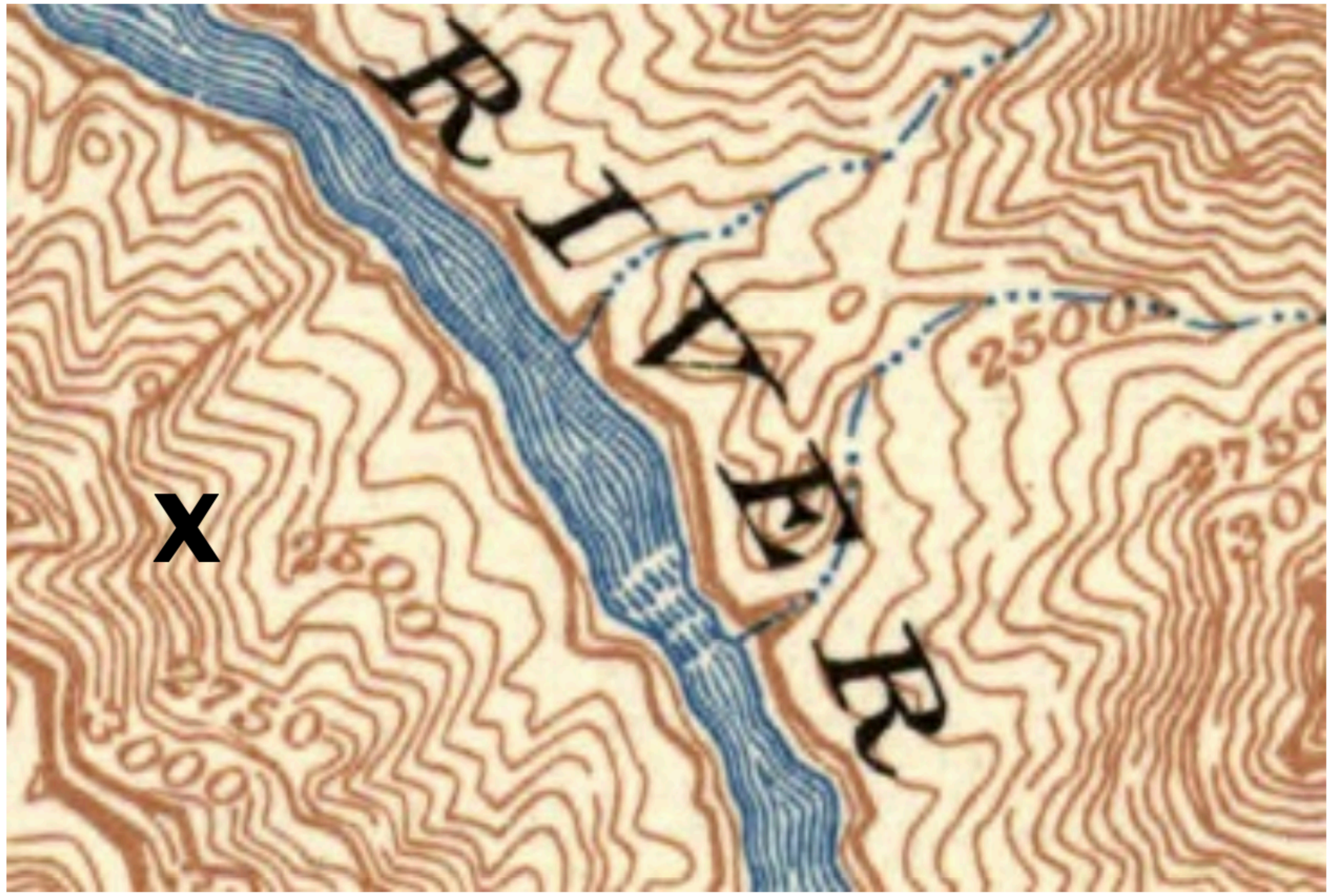

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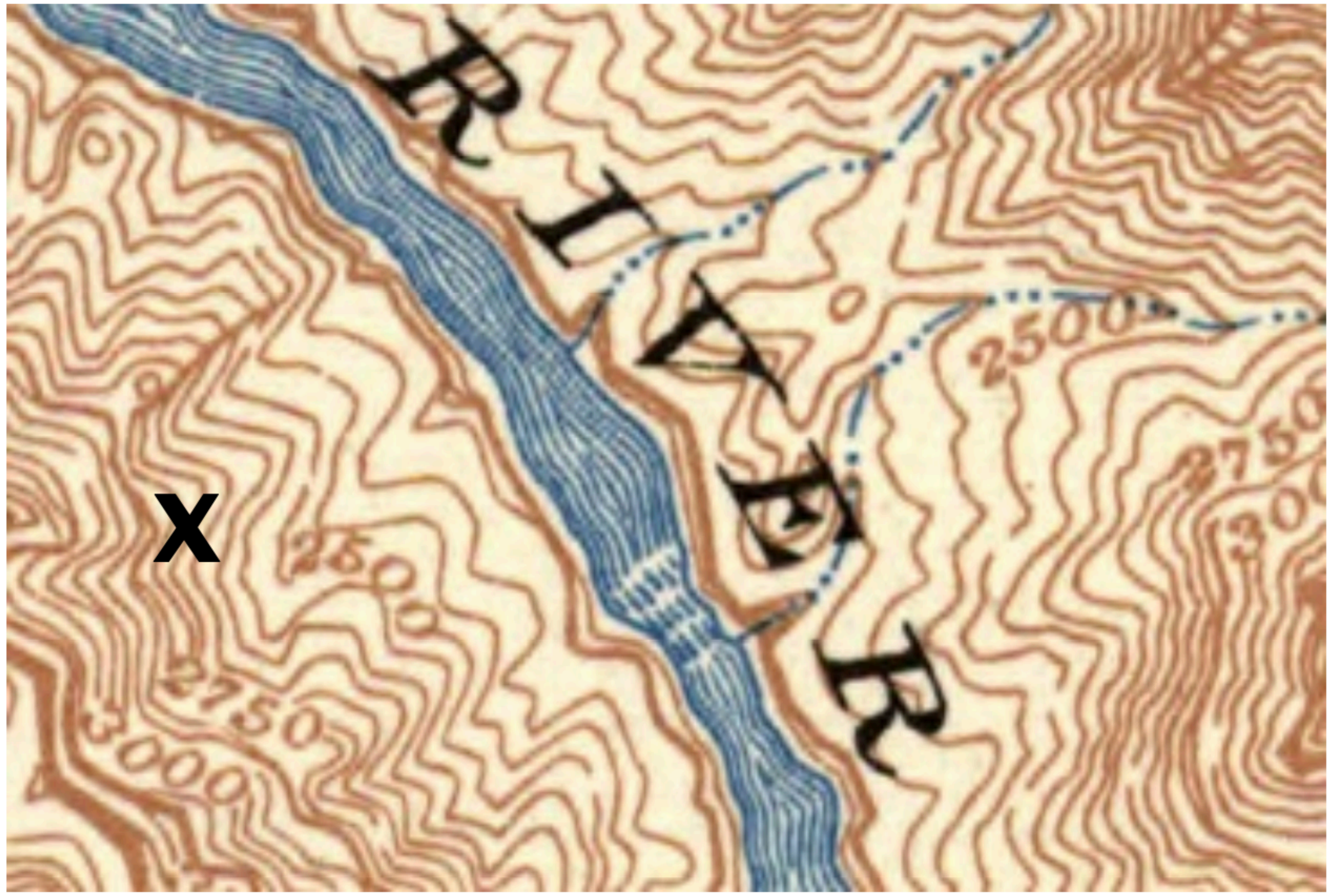

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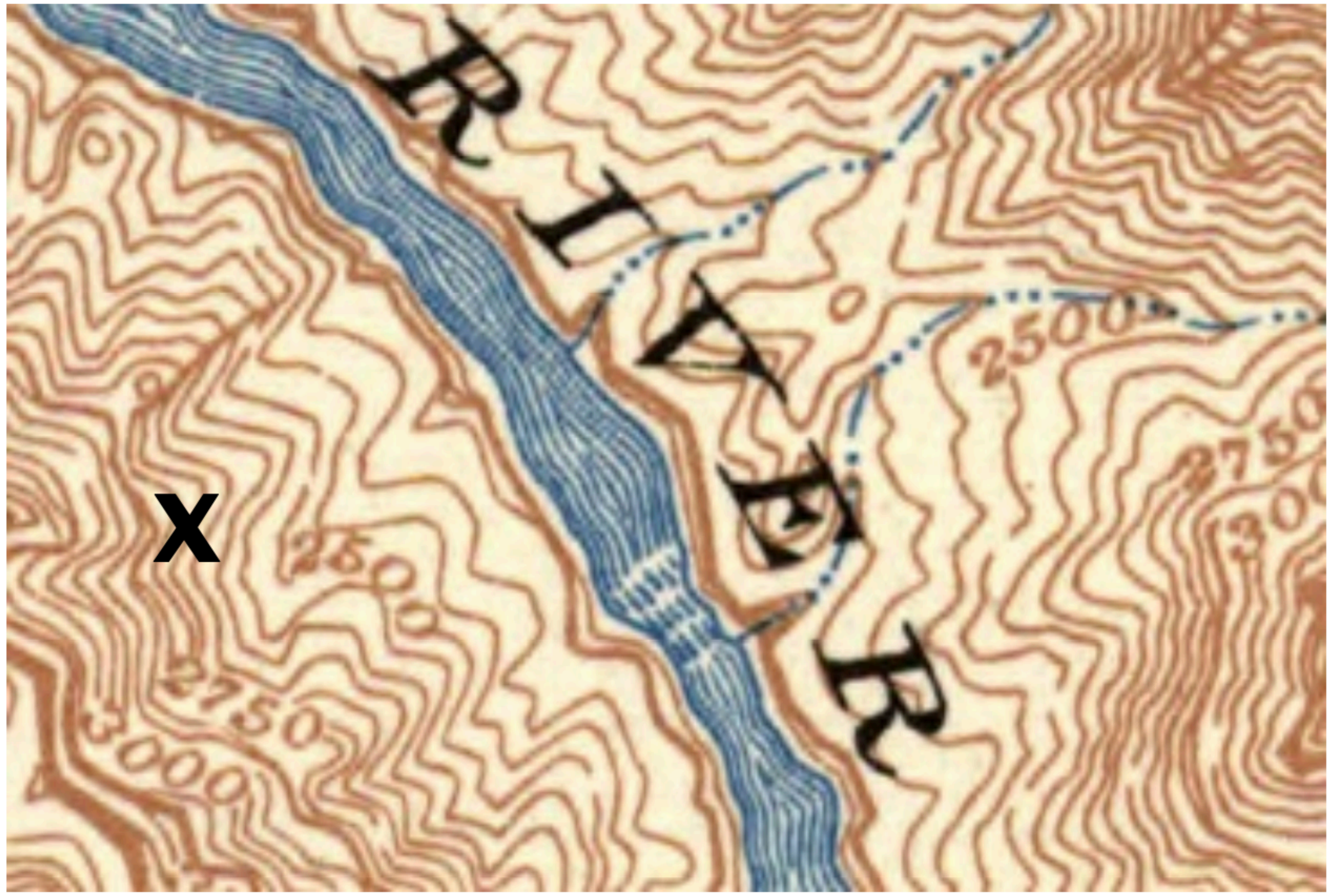


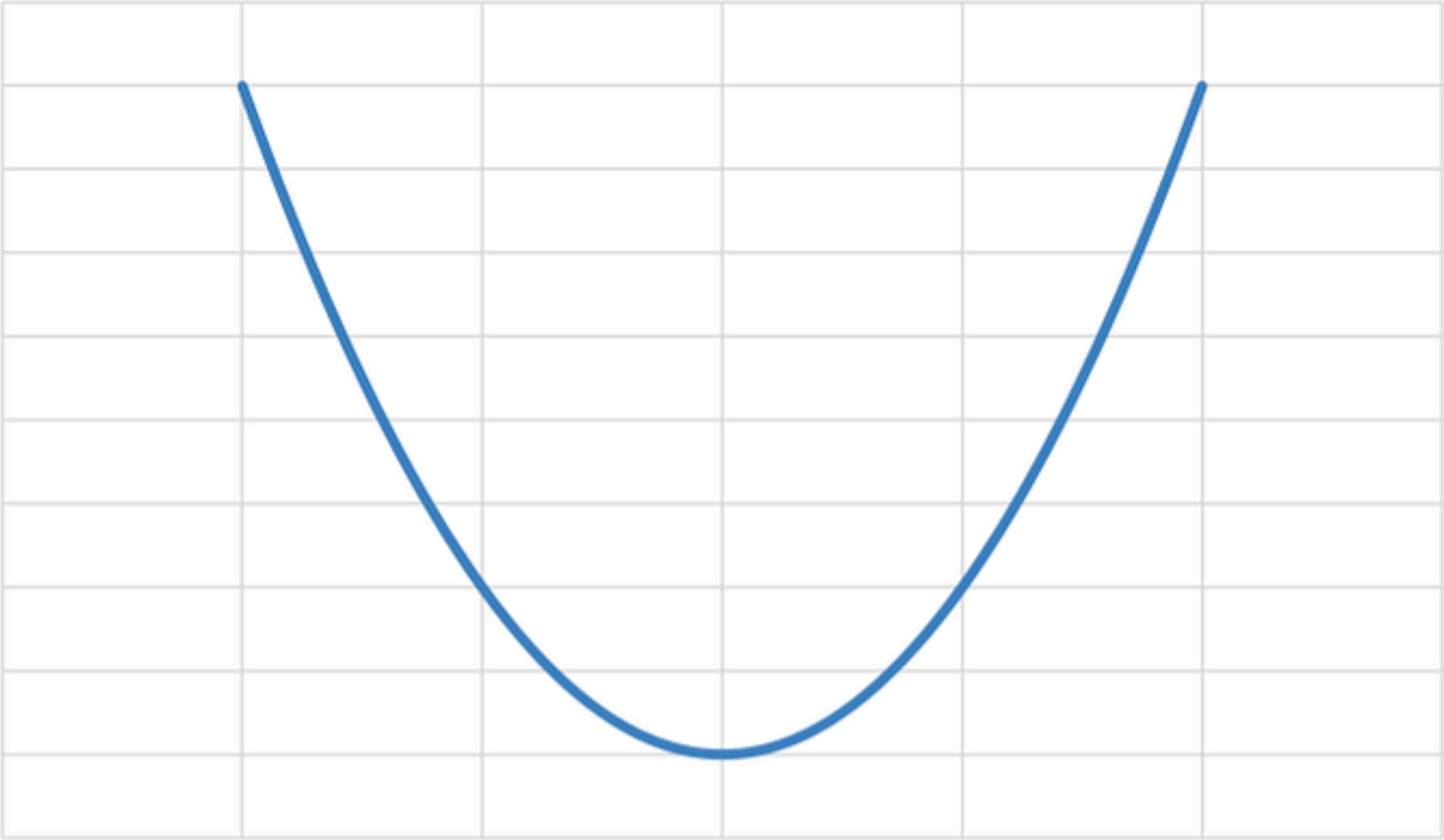
What if multiple equally good alternatives?

Logistic Regression: Loss

Convex function

Image Credit: Medium

Logistic Regression: Loss



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Convex function

- Has only one option for steepest gradient

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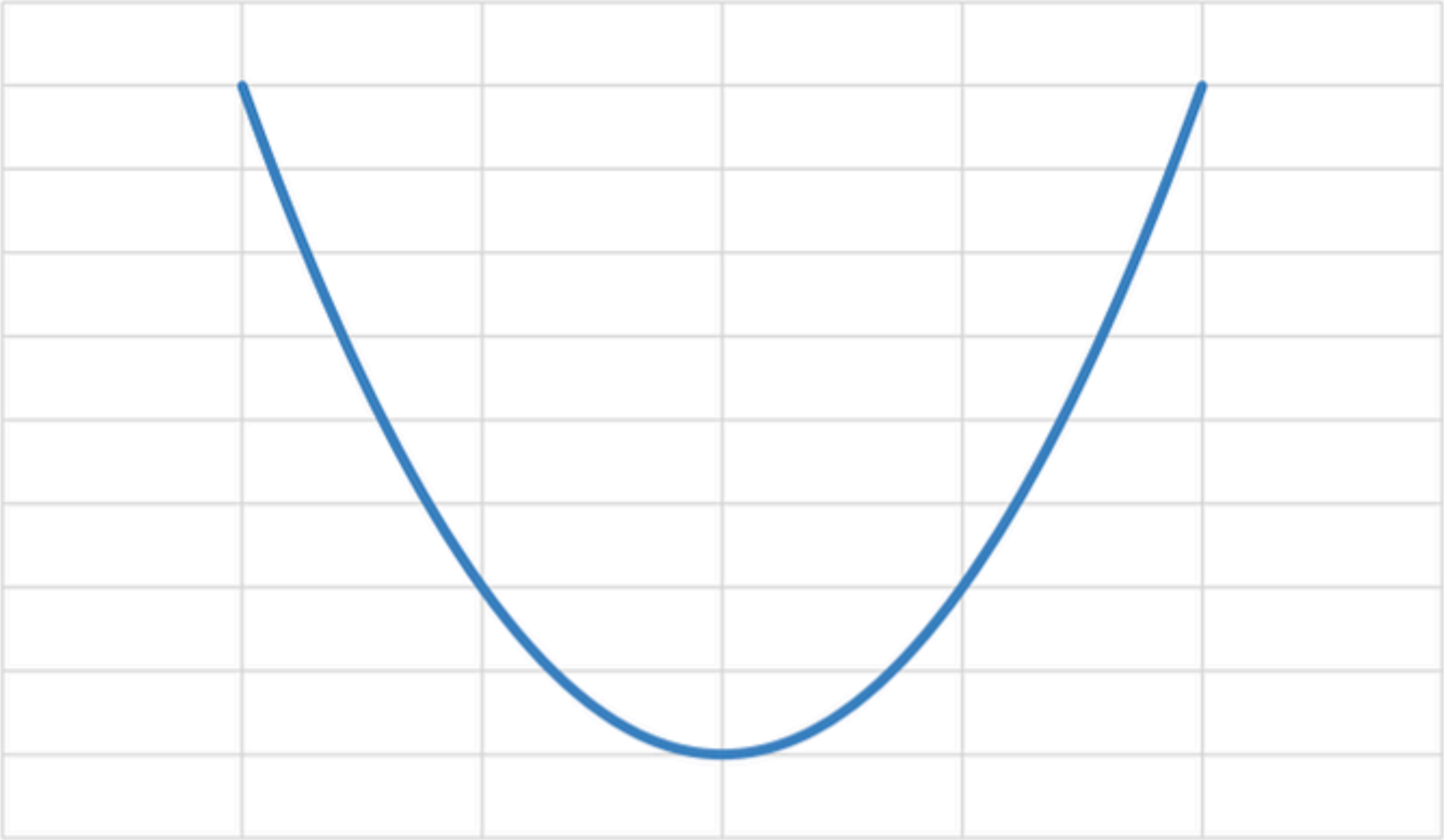

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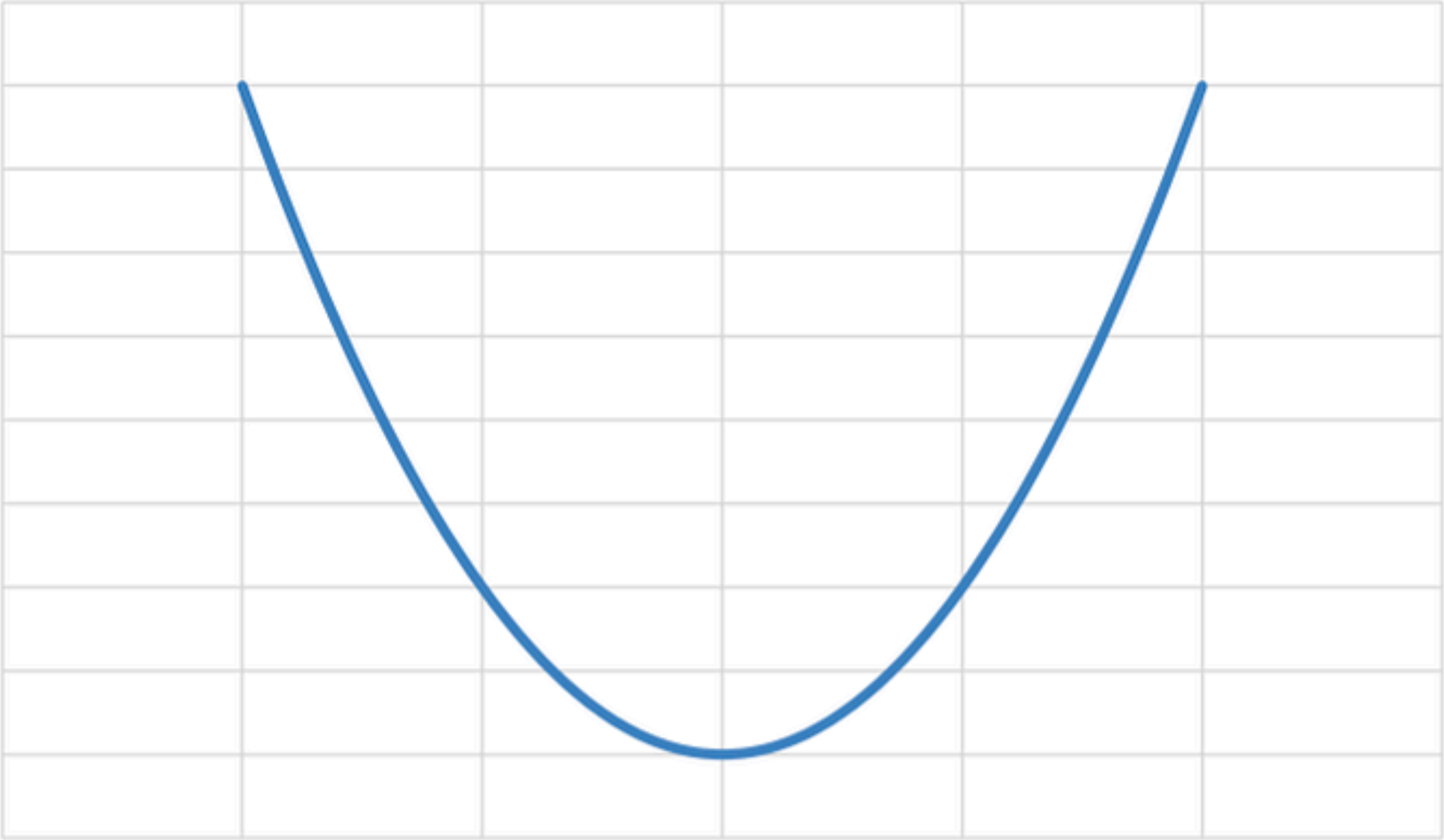

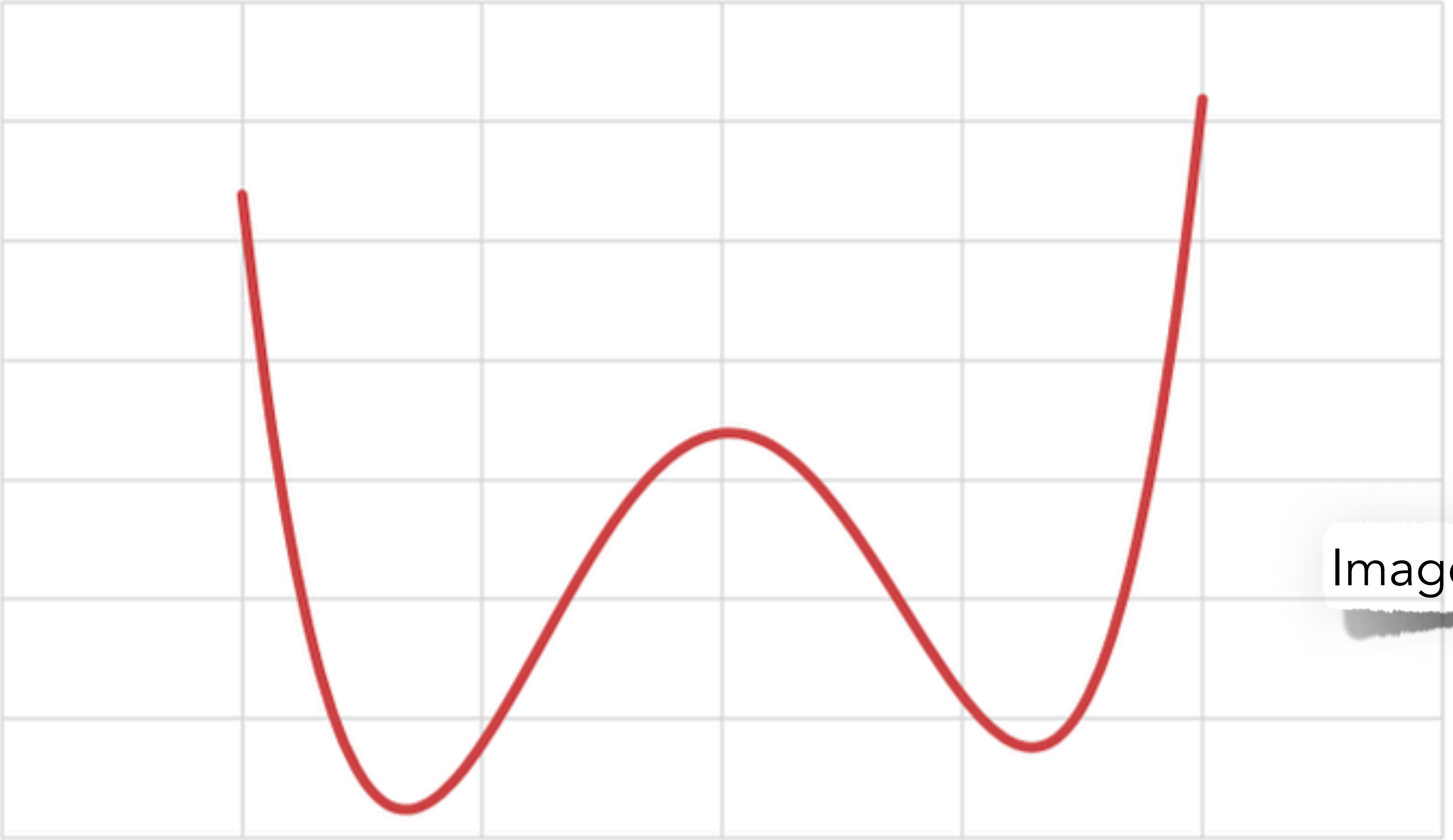

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Convex function

Non-convex function

- Has only one option for steepest gradient
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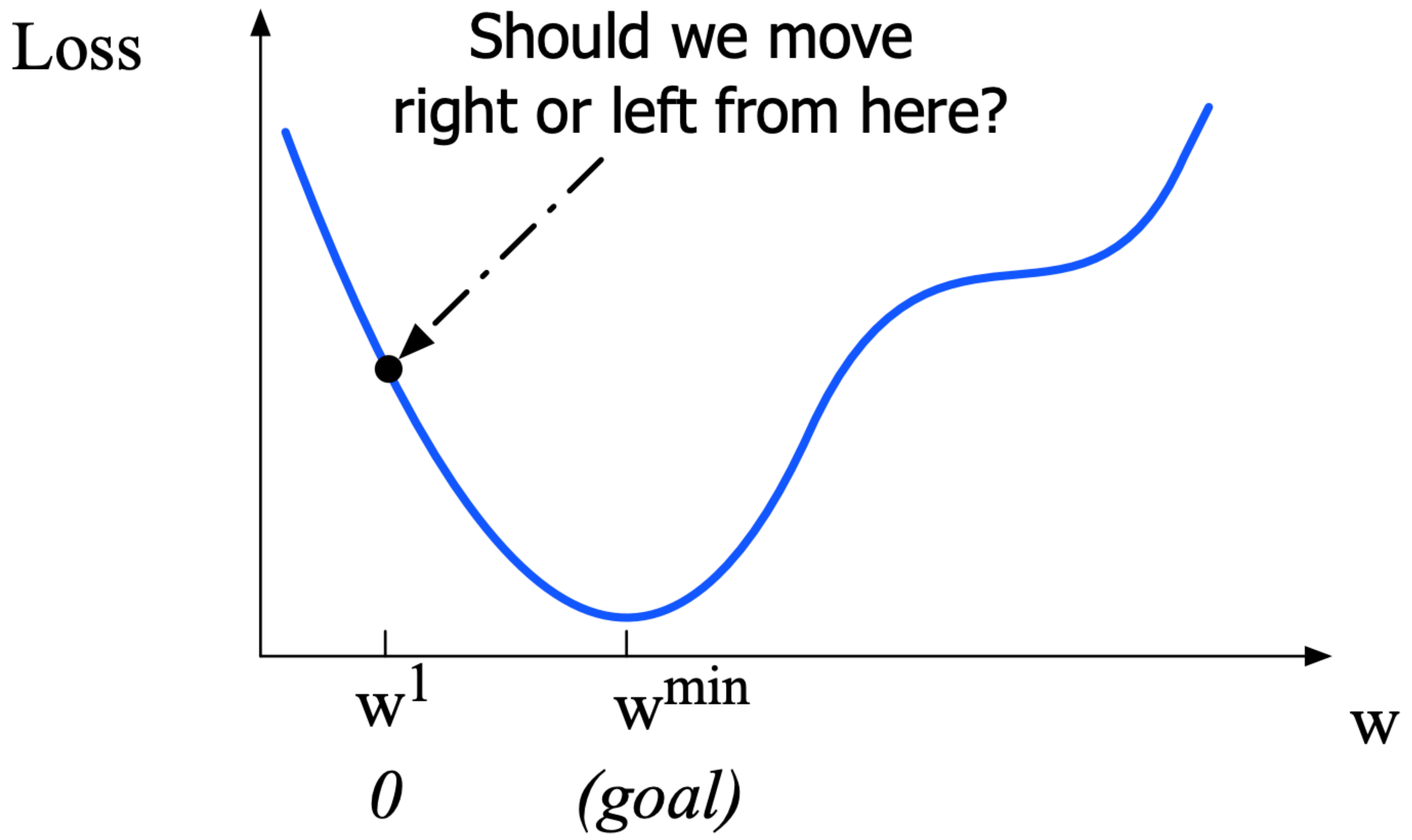
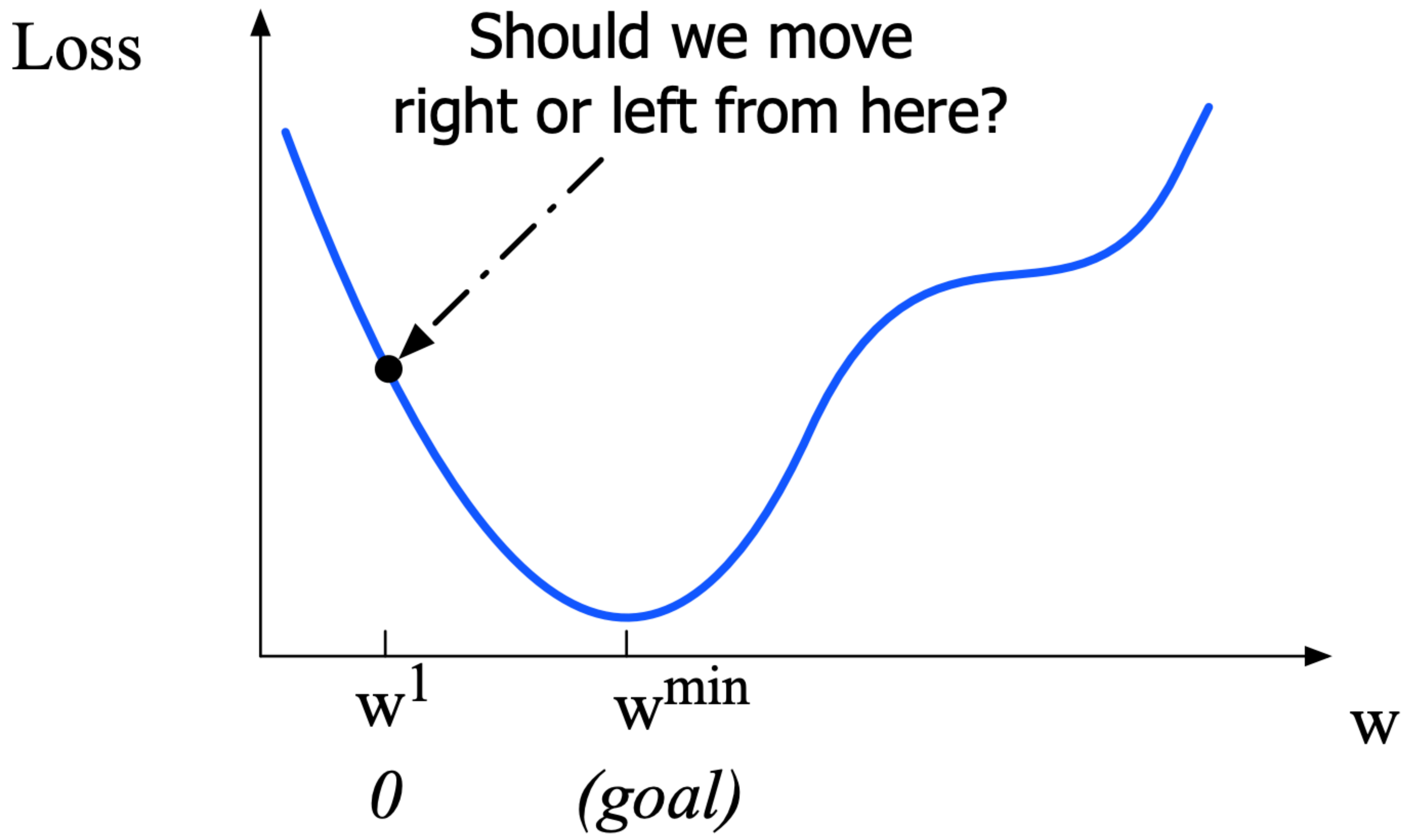
Neural Networks - multiple alternatives

Consider: a single scalar w



Consider: a single scalar w

Given current w, should we make it bigger or smaller?



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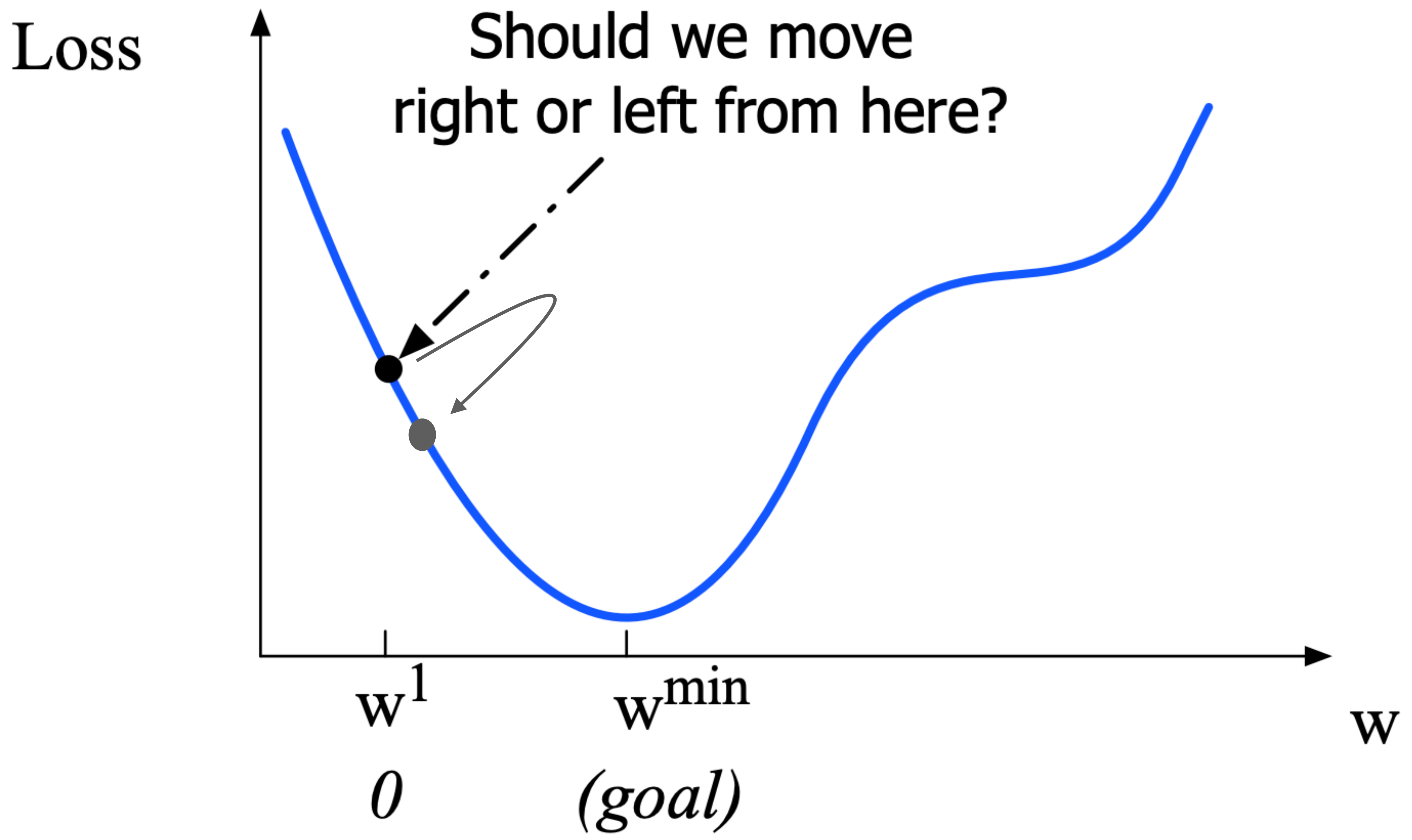

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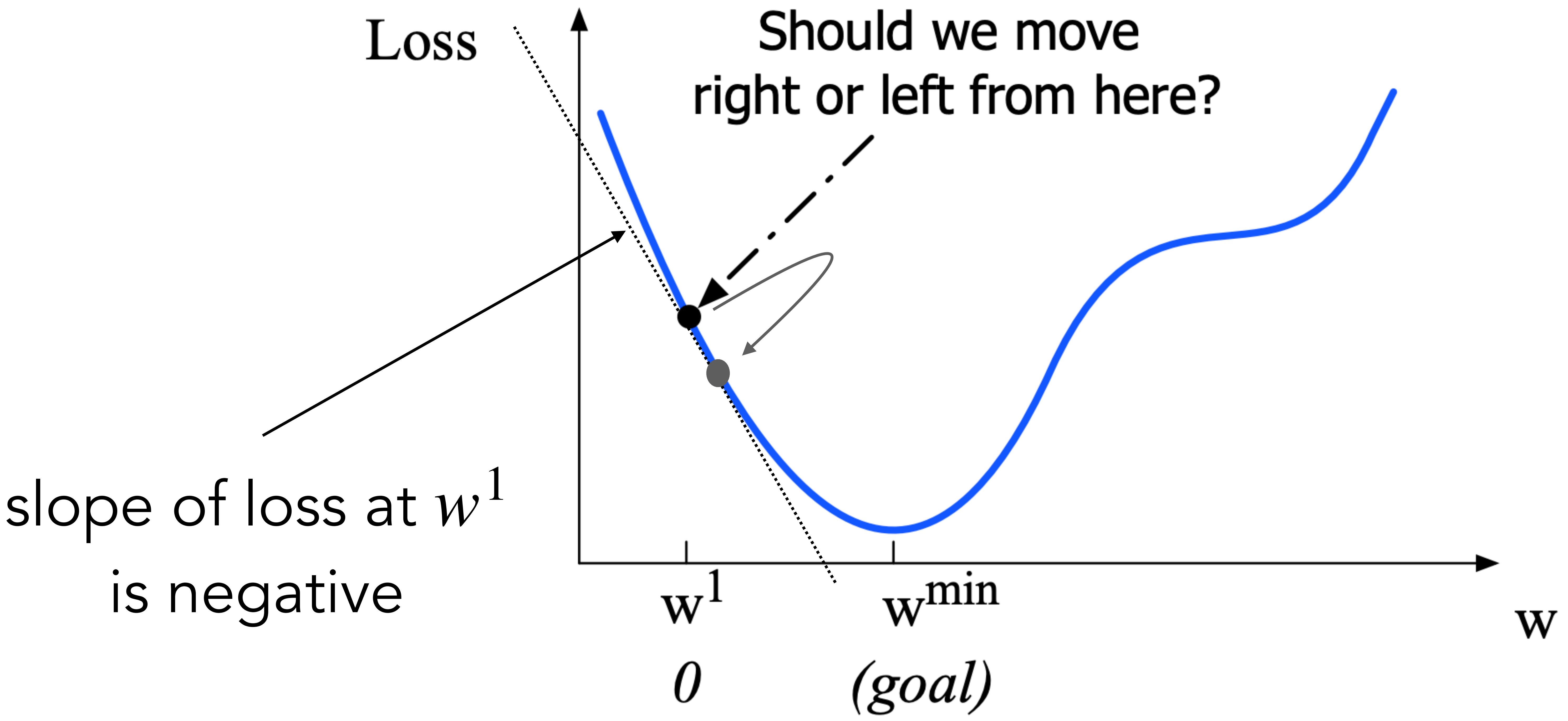

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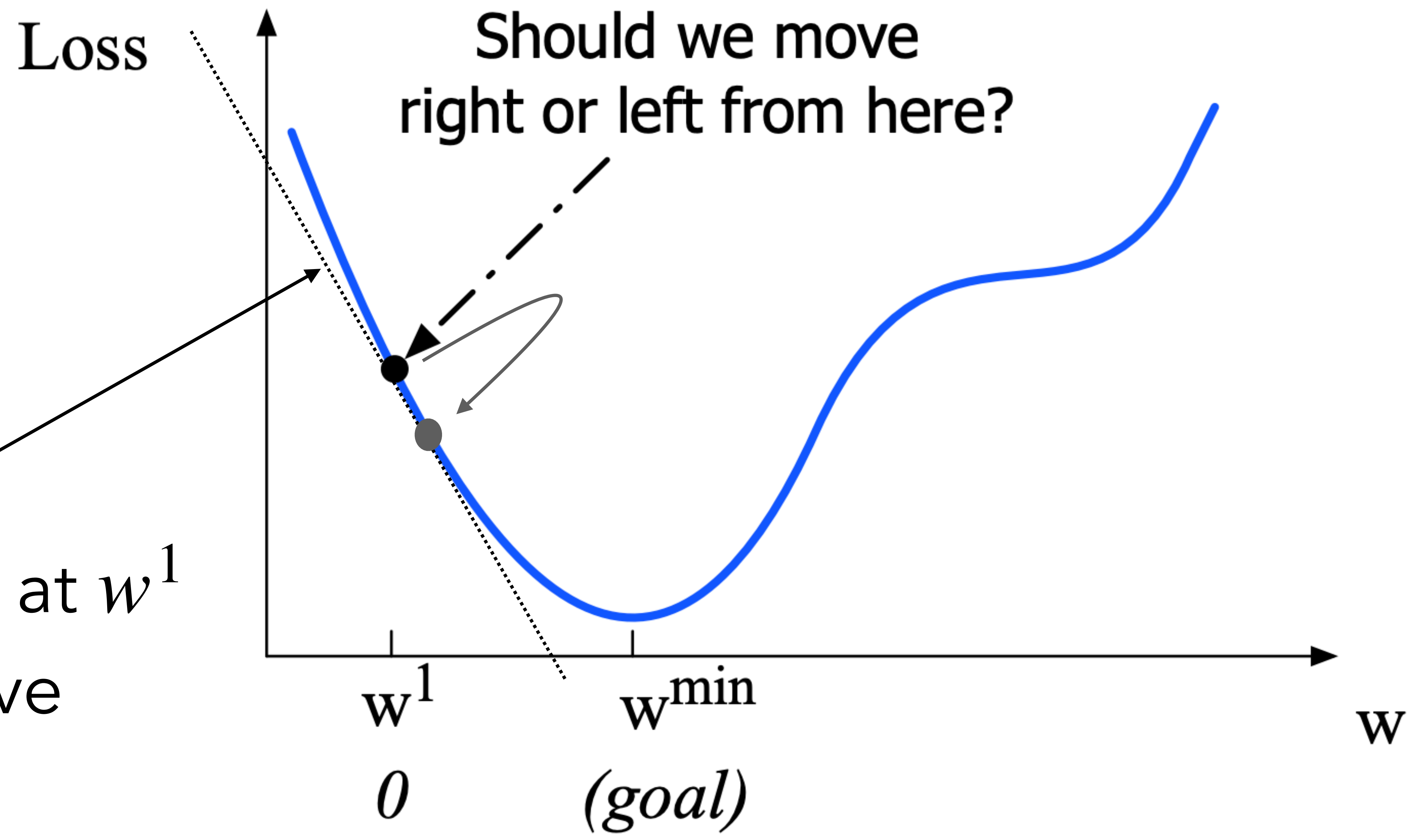


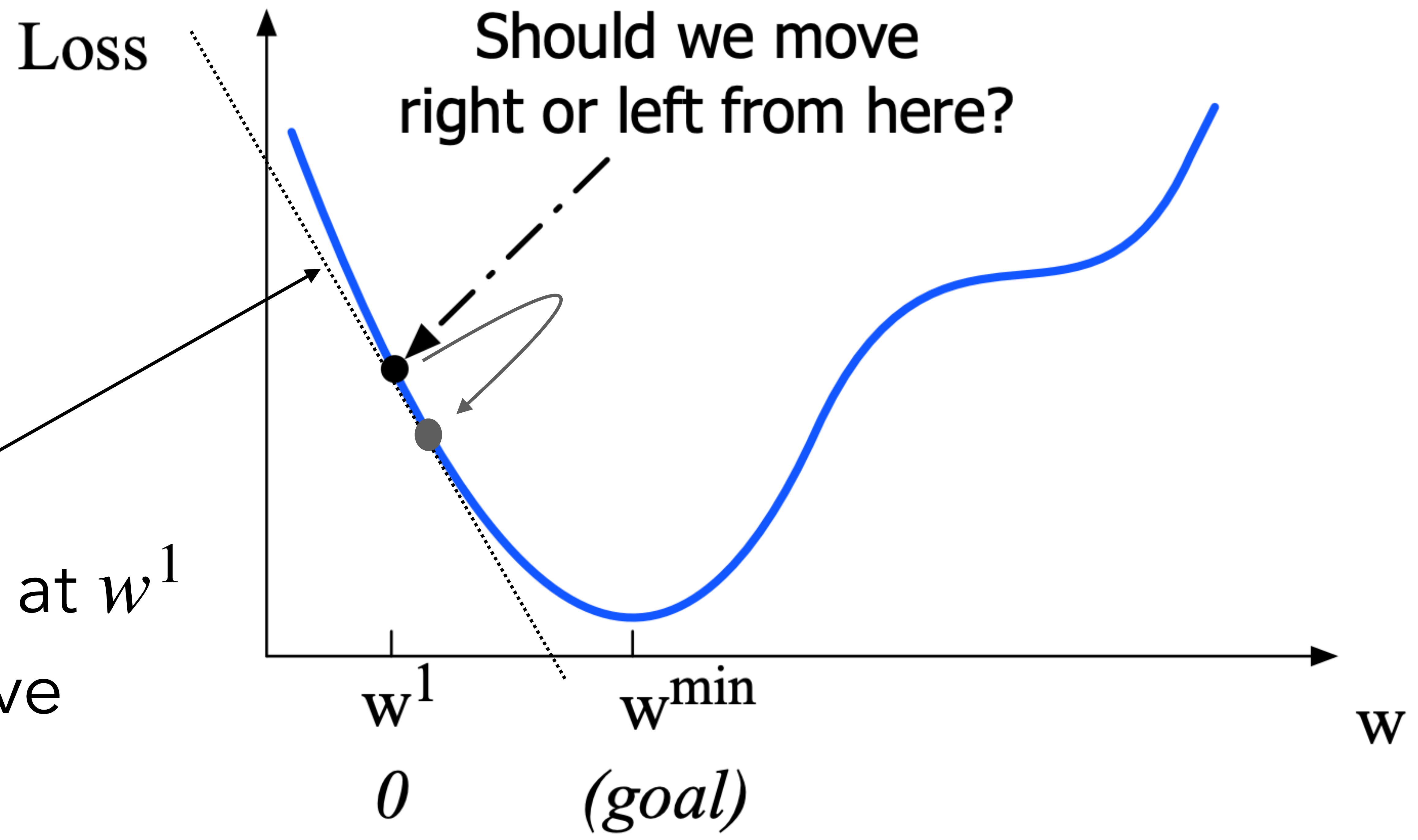
slope of loss at w^1
is negative

need to move positive

Consider: a single scalar w

Given current w, should we make it bigger or smaller?

Move w in the reverse direction from the slope of the function



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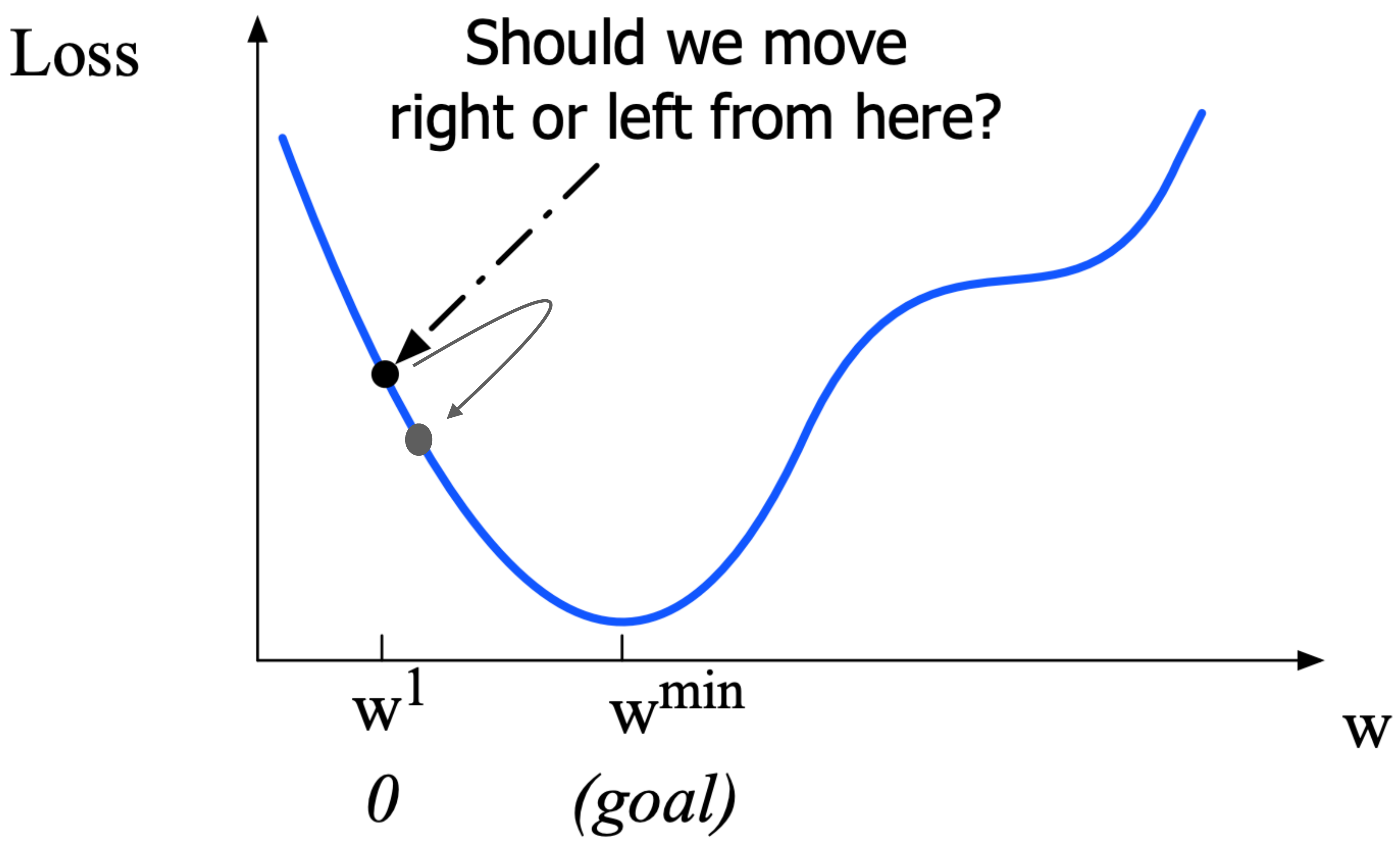
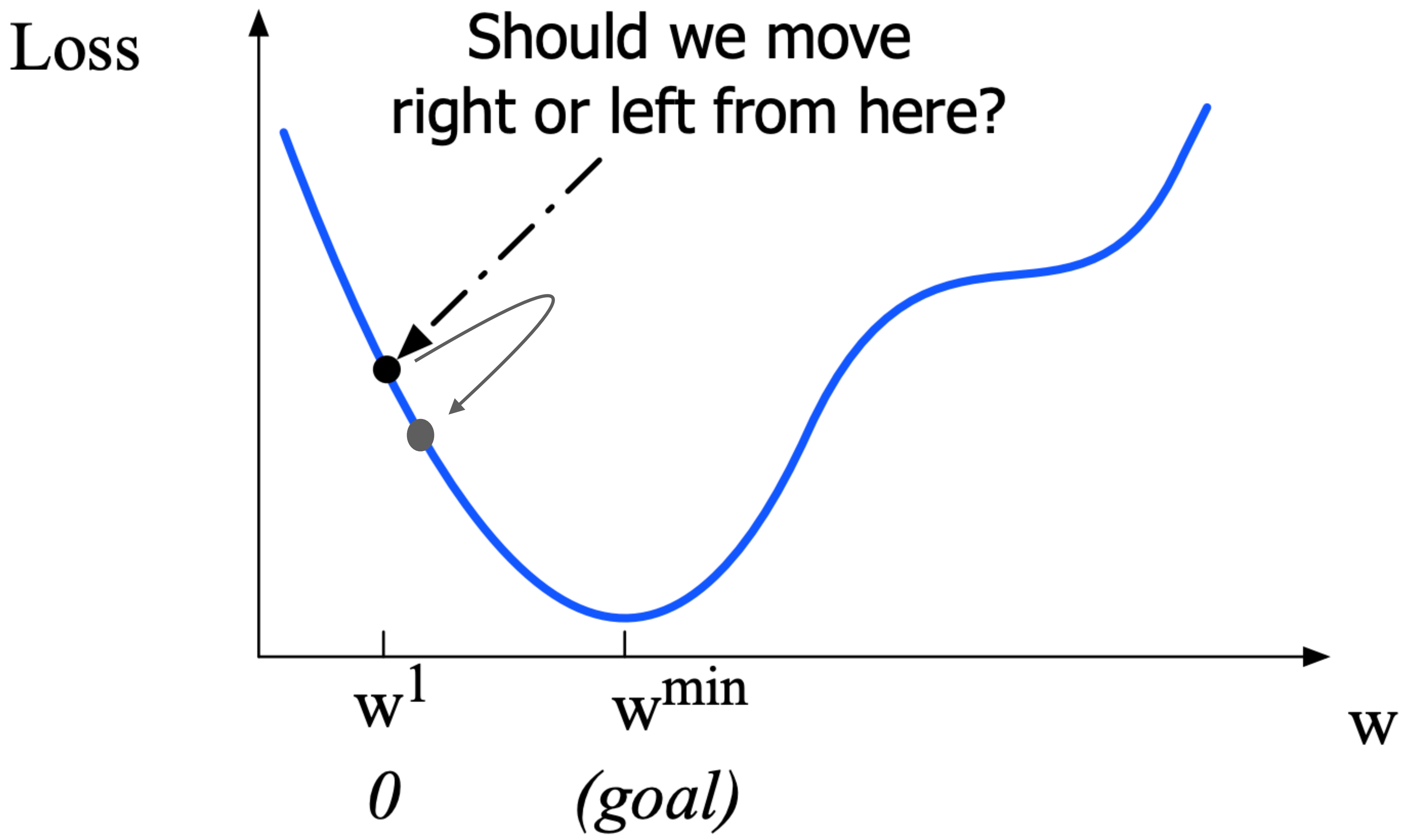
Gradients



Gradients

The **gradient** of a function of many variables is a vector pointing in the direction of the greatest increase in a function.



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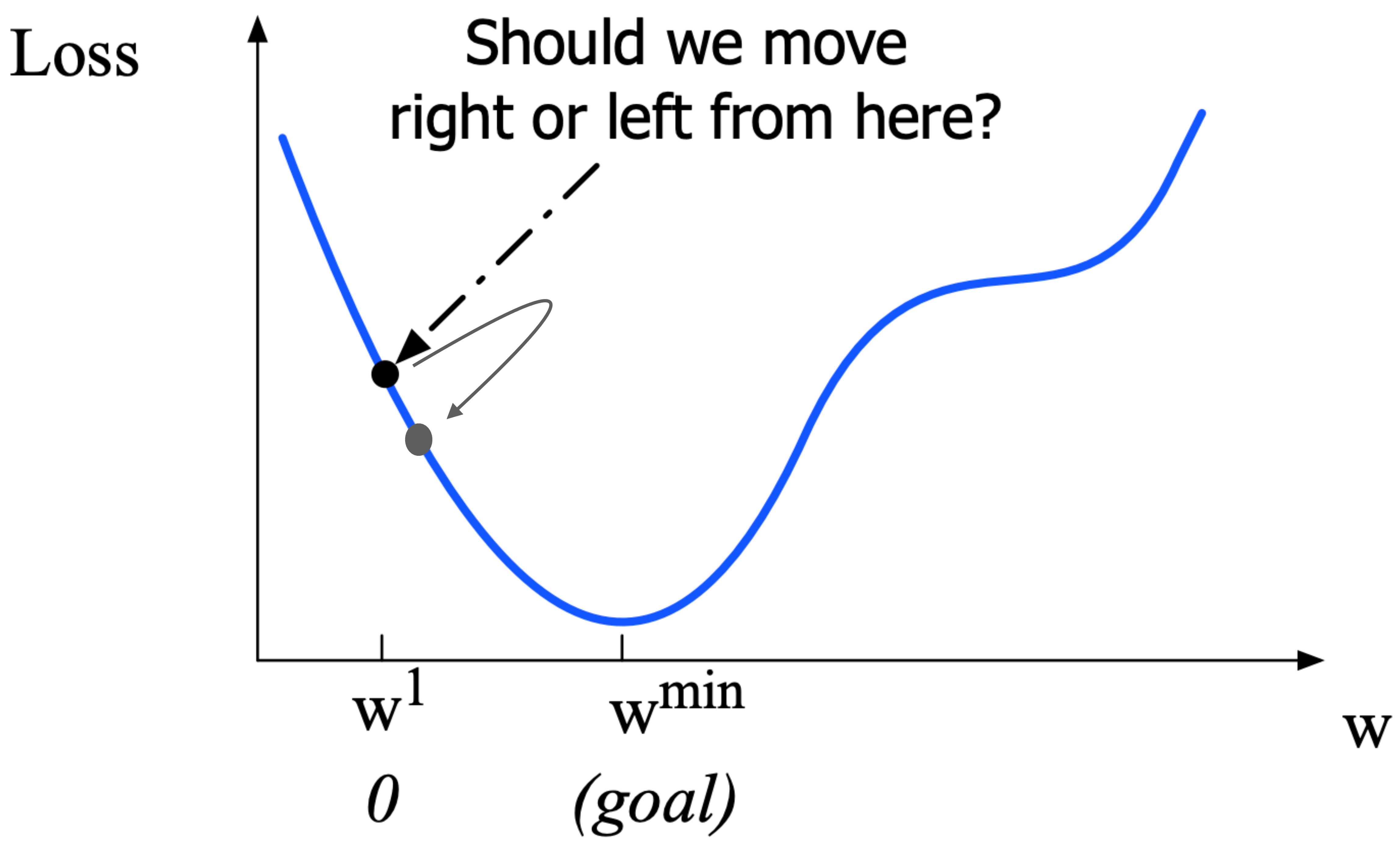

Gradient Descent

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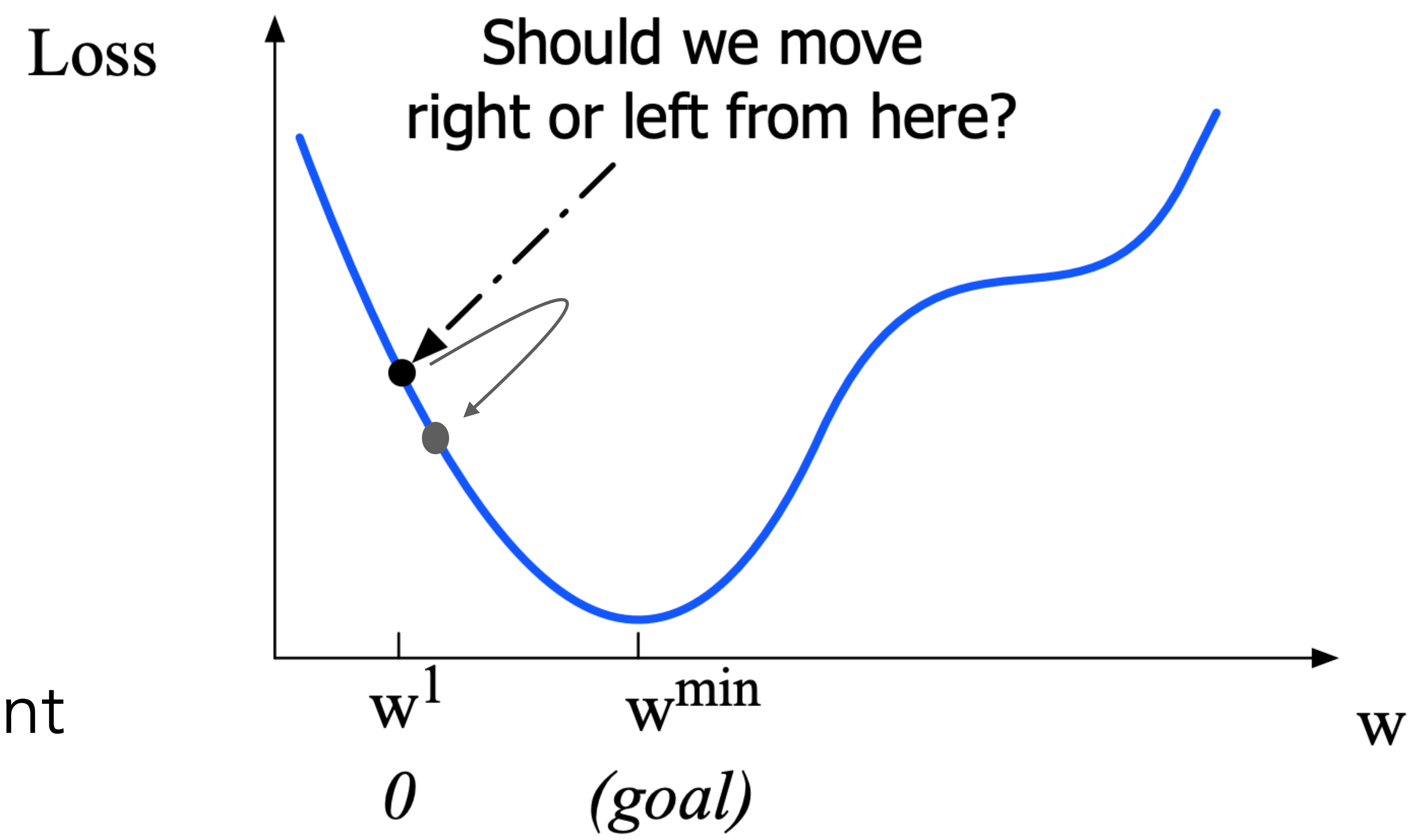
Find the gradient of the loss function at the current point and move in the **opposite** direction.



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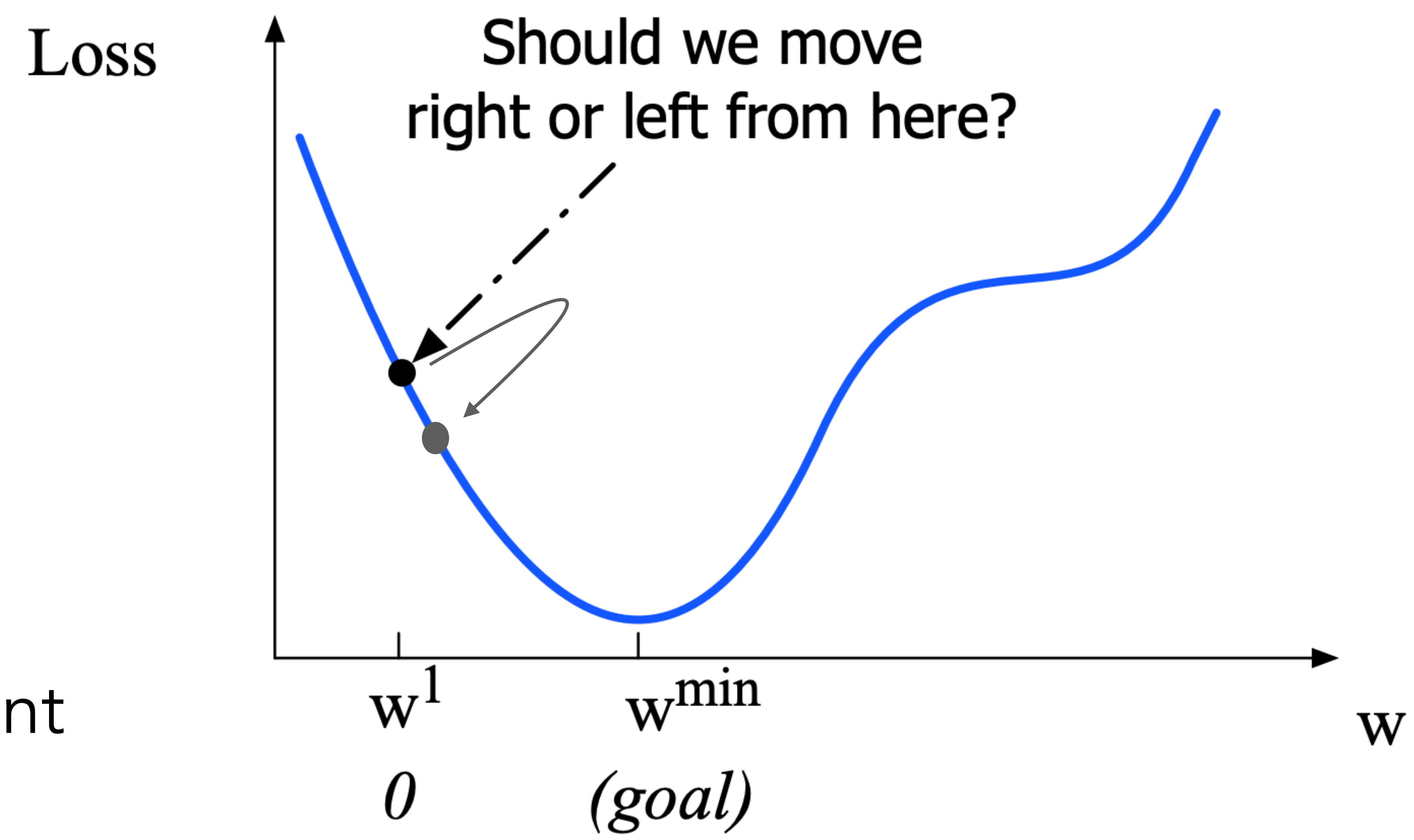

Gradient Descent

But by how much?

Gradient Updates

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- Move w by the value of the gradient $\frac{\partial}{\partial w} L(f(x; w), y^*)$, weighted by a learning rate η

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If parameter θ is a vector of d dimensions:

The gradient is just such a vector; it expresses the directional components of the sharpest slope along each of the d dimensions.

Under 2 dimensions

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Consider 2 dimensions, w and b:

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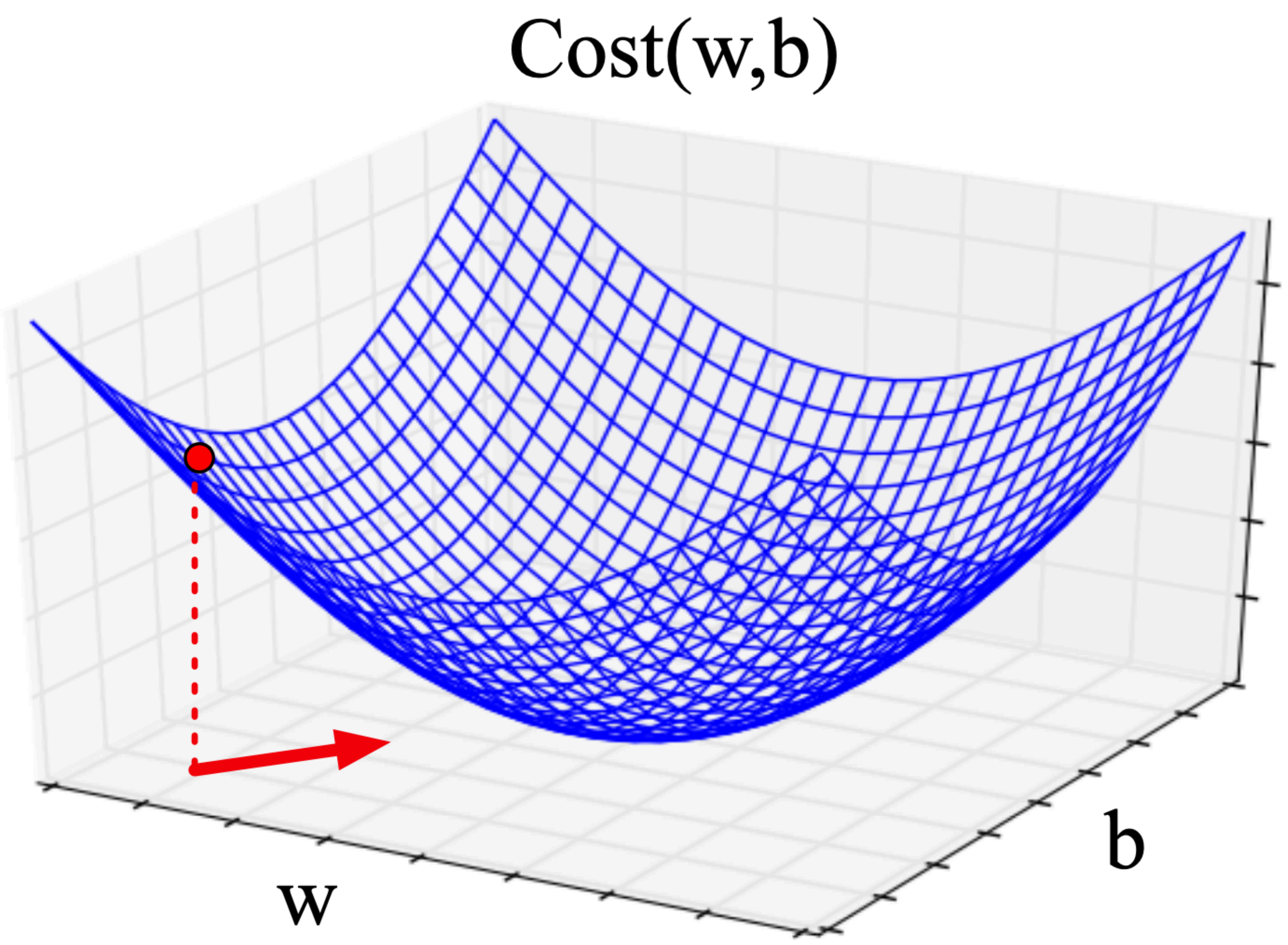
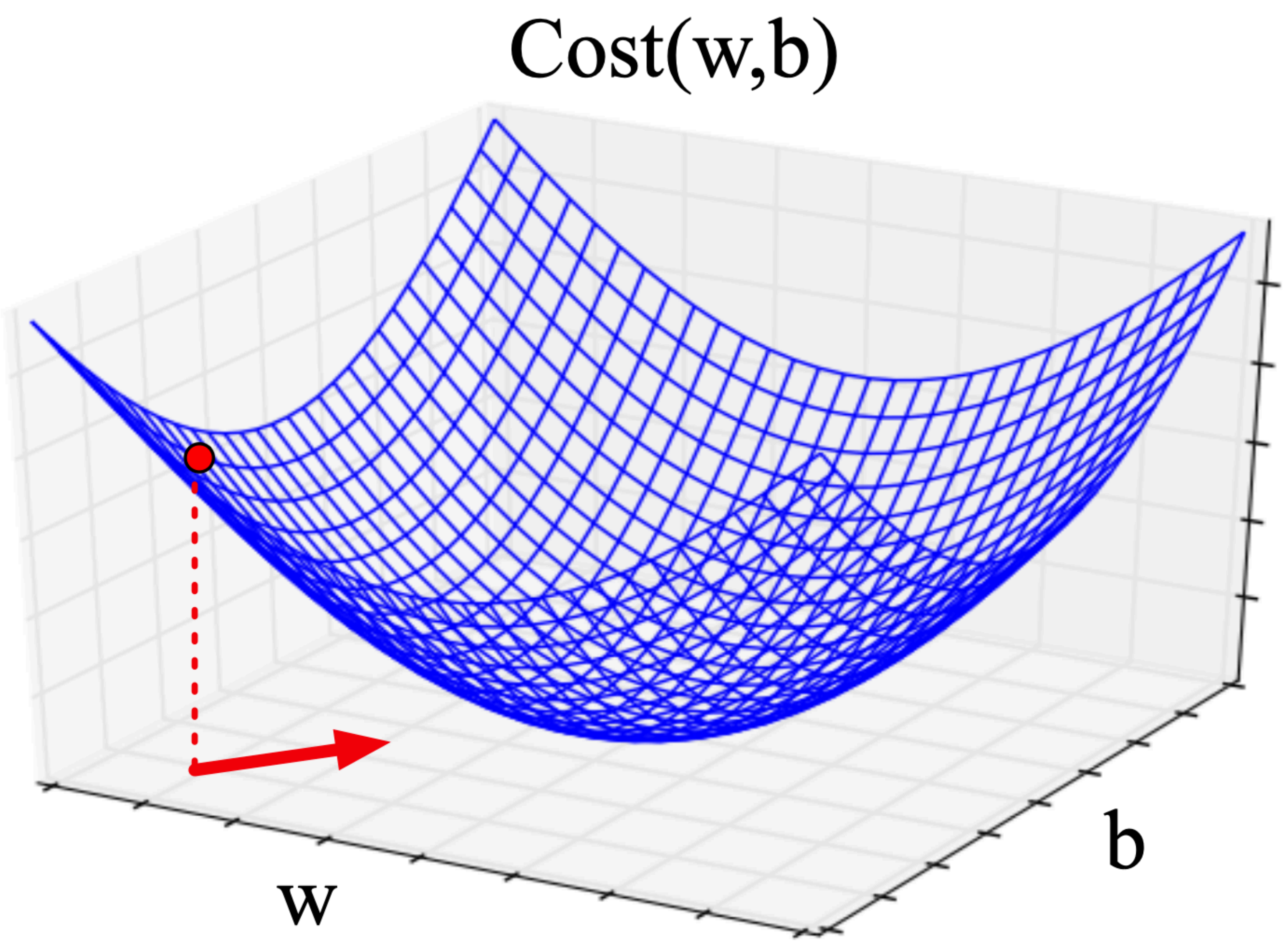


Under 2 dimensions

Consider 2 dimensions, w and b:

Visualizing the gradient vector at the red point

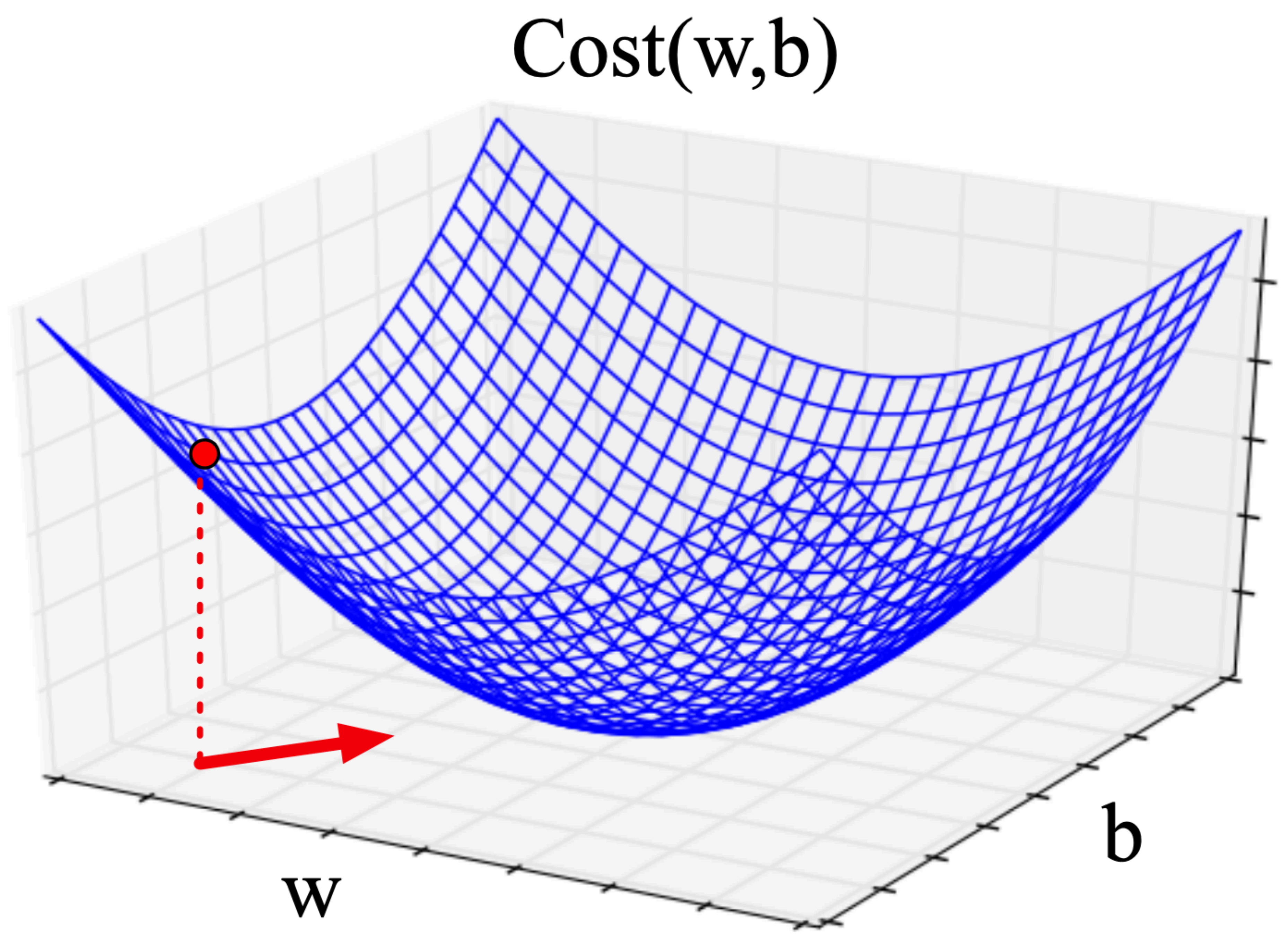


Under 2 dimensions

Consider 2 dimensions, w and b:

Visualizing the gradient vector at the red point

It has two dimensions shown in the $x - y$ plane



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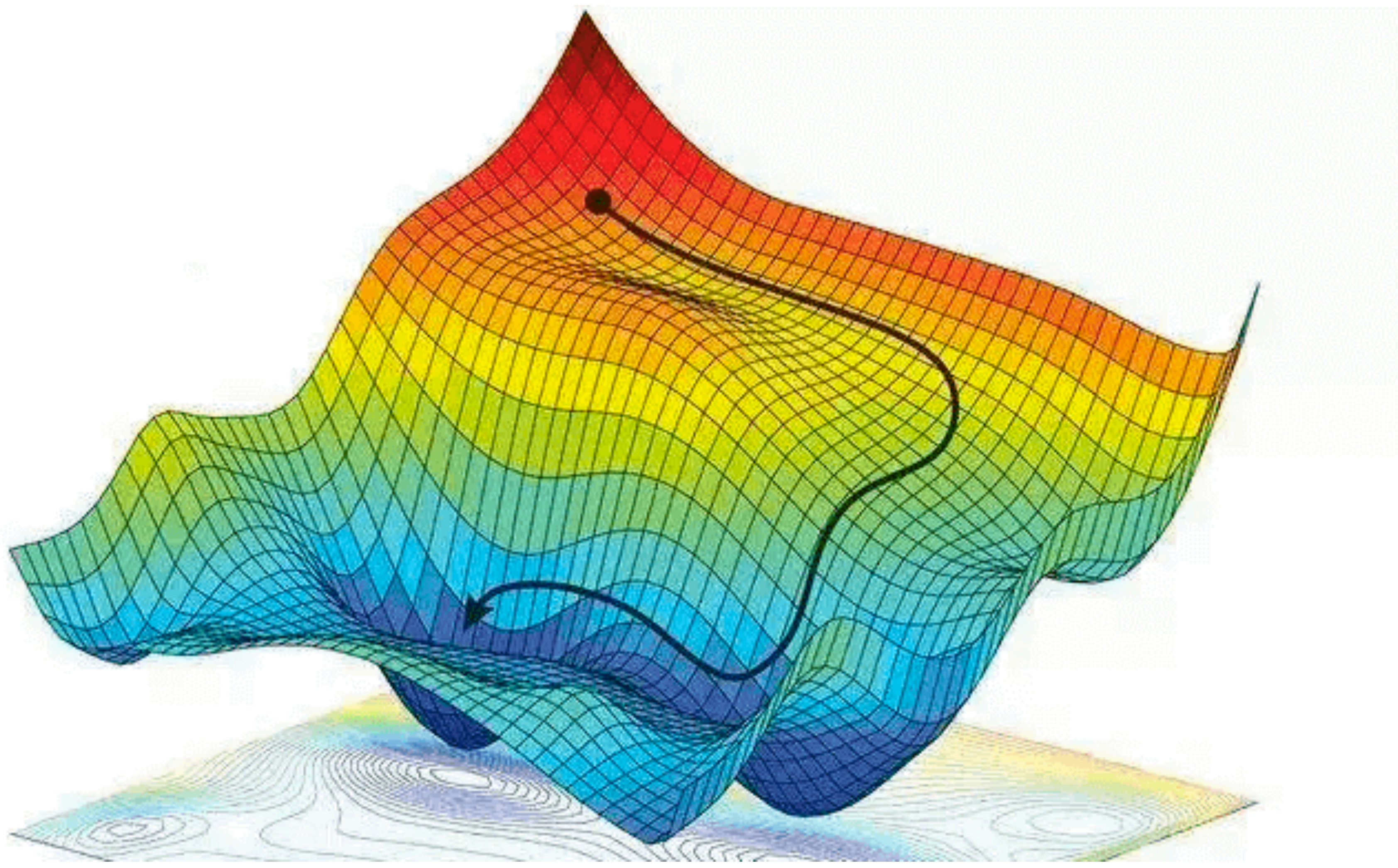

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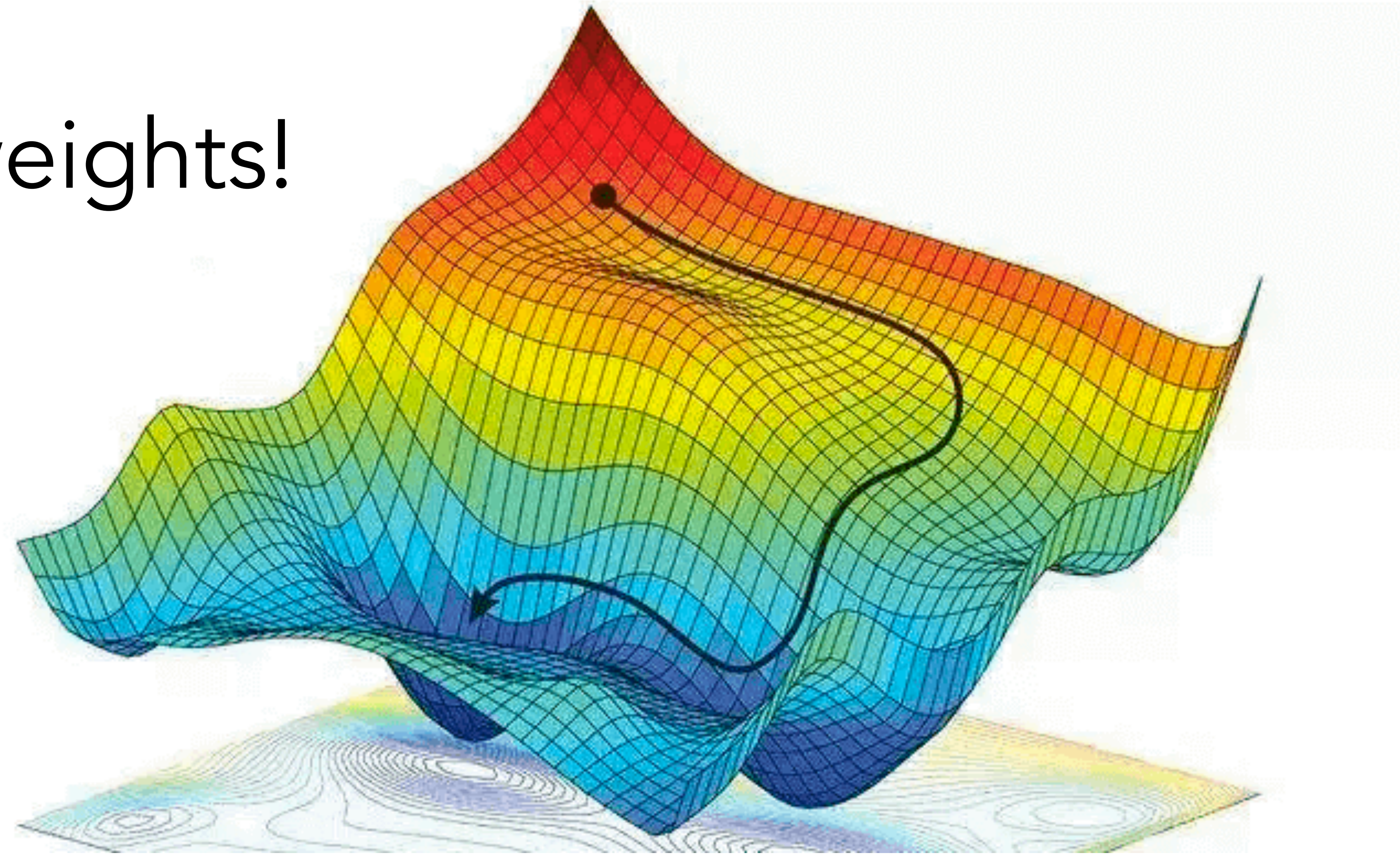

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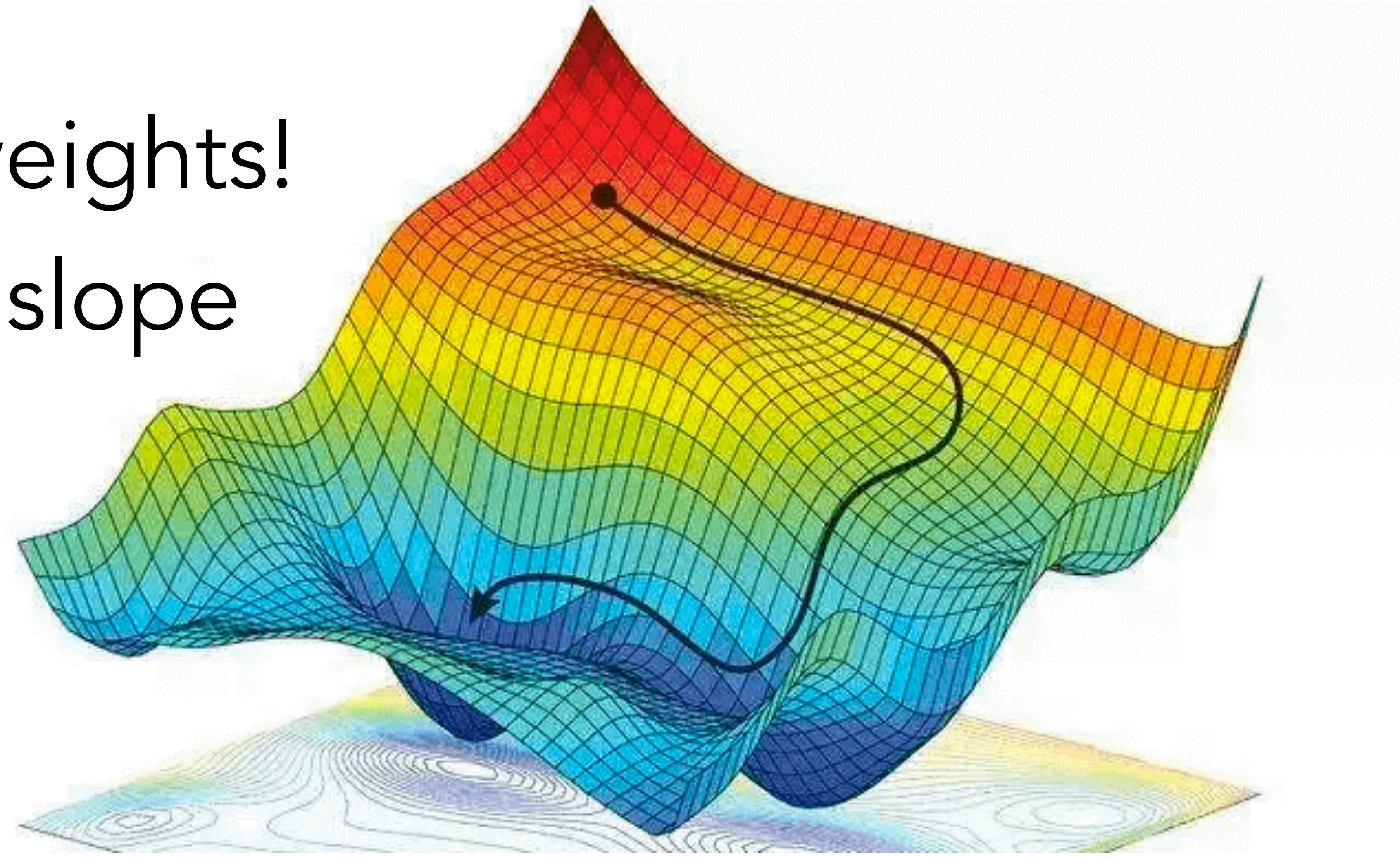

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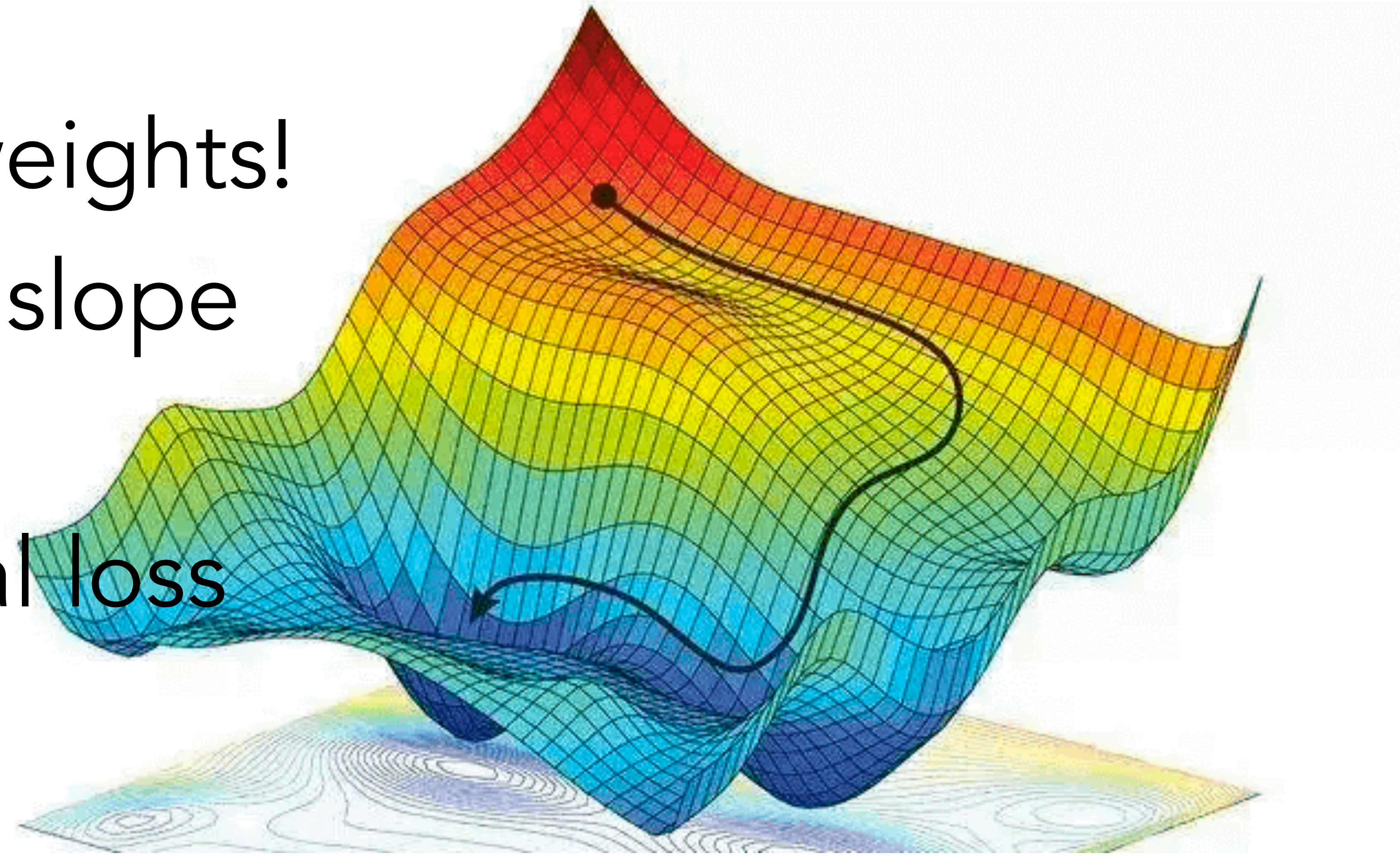

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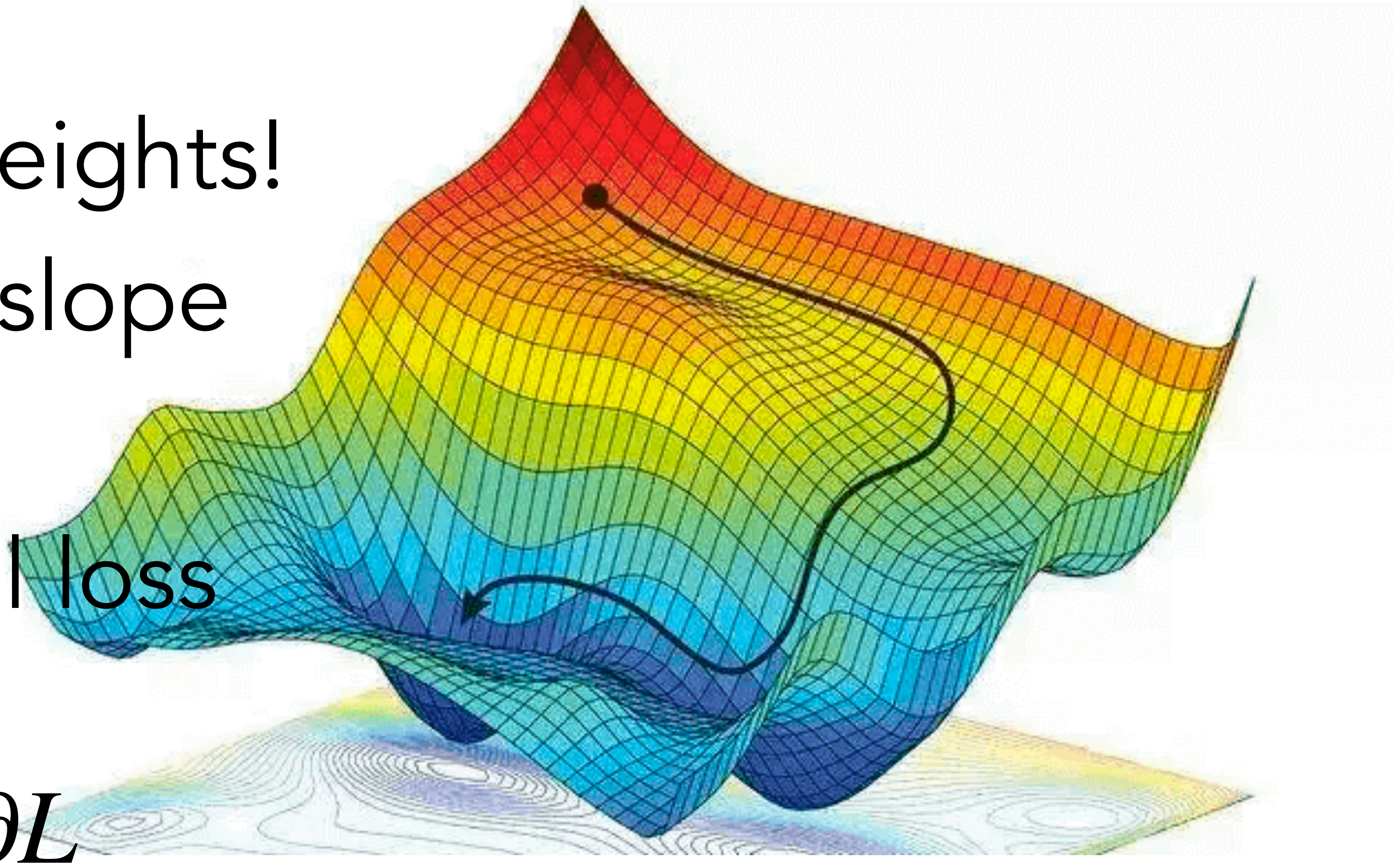

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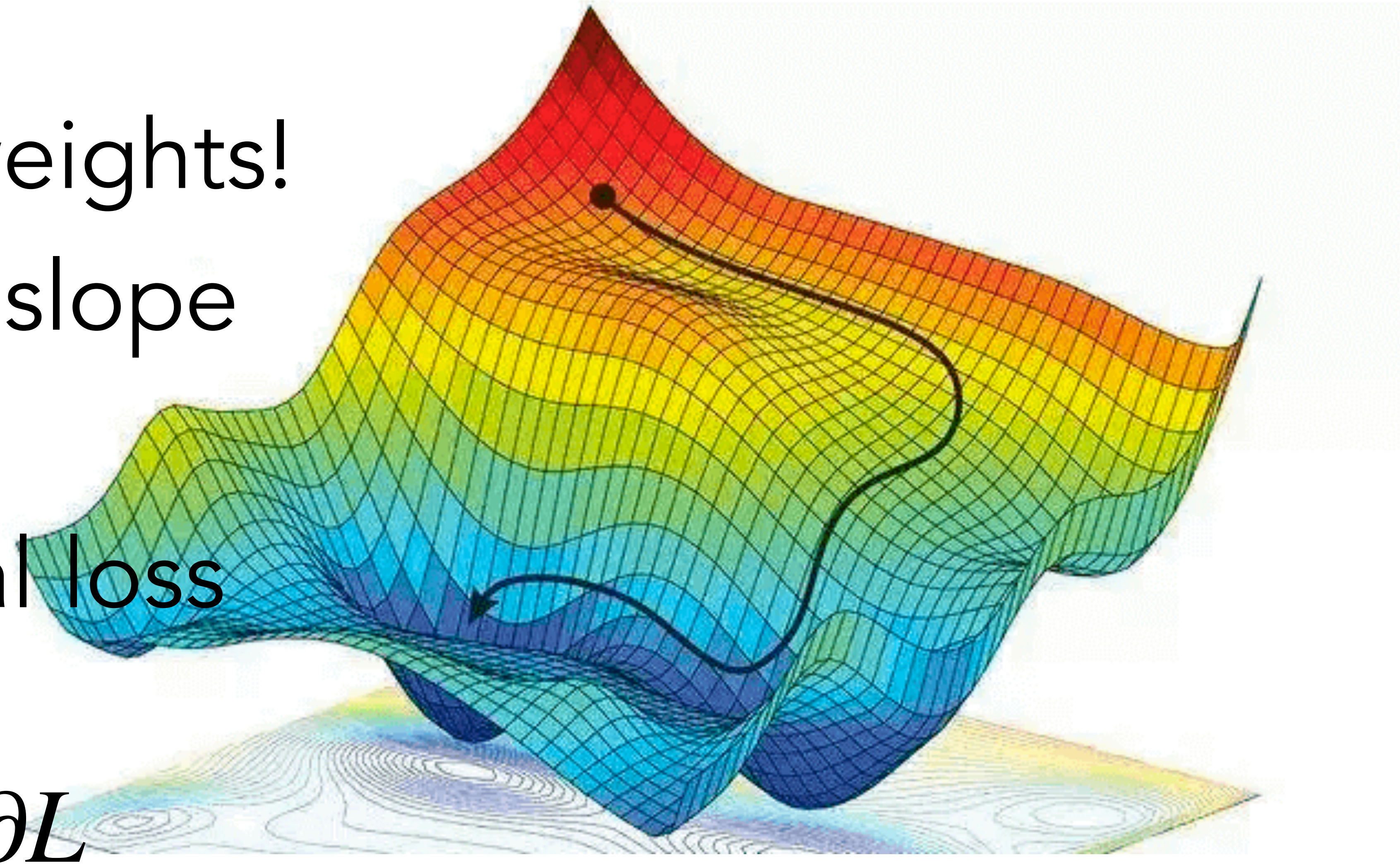

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- For each dimension θ_i the gradient component i tells us the slope with respect to that variable
  - "How much would a small change in θ_i influence the total loss function L?"
- We express the slope as a partial derivative $\frac{\partial}{\partial \theta_i}$ of the loss, $\frac{\partial L}{\partial \theta_i}$
  - The gradient is then defined as a vector of these partials

Real-life gradients

We will represent $\hat{y}$ as $f(x;\theta)$ to make the dependence on θ more obvious

$$\nabla_\theta L(f(x;\theta),y)) = \begin{bmatrix} \frac{\partial}{\partial w_1} L(f(x;\theta),y) \\ \frac{\partial}{\partial w_2} L(f(x;\theta),y) \\ \vdots \\ \frac{\partial}{\partial w_n} L(f(x;\theta),y) \end{bmatrix}$$

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The final equation for updating θ at time step $t+1$ based on the gradient is thus:

$$\theta_{t+1} = \theta_t - \eta \frac{\partial}{\partial \theta} L(f(x;\theta), y)$$

Gradients for Logistic Regression

Case: Sentiment Analysis

Recall: the cross-entropy loss for logistic regression

$$L_{CE}(y, \hat{y}) = -\left[y \log \sigma(\mathbf{w} \cdot \mathbf{x} + b) + (1 - y)\log(\sigma(-\mathbf{w} \cdot \mathbf{x} + b))\right]$$

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Derivatives have a closed form solution:

$$\frac{\partial L_{CE}(y, \hat{y})}{\partial w_j} = [\sigma(\mathbf{w} \cdot \mathbf{x} + b) - y]x_j$$

Pseudocode

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function STOCHASTIC GRADIENT DESCENT $(L(), f(), x, y)$ returns θ

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where: L is the loss function

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Stochastic Gradient Descent

Mini-Batching

function STOCHASTIC GRADIENT DESCENT $(L(), f(), x, y, m)$ returns θ

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y is the set of training outputs (labels) $y^{(1)}, y^{(2)}, \ldots y^{(N)}$ and m is the mini-batch size

$\theta \leftarrow 0$ (or randomly initialized)

repeat till done

for each randomly sampled minibatch of size m:

1. for each training tuple$(\mathbf{x}^{(i)}, y^{(i)})$ in the minibatch: (in random order)
   i. Compute $\hat{y}^{(i)} = f(\mathbf{x}^{(i)}; \theta)$ # What is our estimated output $\hat{y}^{(i)}$?
   ii. Compute the loss $L_{mini} \leftarrow L_{mini} + L(\hat{y}^{(i)}, y^{(i)})$ # How far off is $\hat{y}^{(i)}$ from the true output $y^{(i)}$?
2. $g \leftarrow \frac{1}{m} \nabla L_{mini}(f(\mathbf{x}^{(i)}; \theta), y^{(i)})$ # How should we move θ to maximize loss?
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Why is this better than stochastic gradient descent?

Lecture Outline

- Recap
  - Smoothing
  - Basics of Supervised Machine Learning
    - Data: Preprocessing and Feature Extraction
- Quiz
- Announcements
- Basics of Supervised Machine Learning
  I. Data: Preprocessing and Feature Extraction
  II. Model:
      I. Logistic Regression
  III. Loss
  IV. Optimization Algorithm
  V. Inference

Regularization

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Why?

Overfitting

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A good model should be able to generalize

What happens when a feature only occurs with one class?

e.g. word “wow” for positive reviews

USC Viterbi

Overfitting: Features

This movie drew me in, and it'll do the same to you.

I can't tell you how much I hated this movie. It sucked.

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4-gram features that just "memorize" training set and might cause problems

x_5 = **"the same to you"**
x_6 = **"tell you how much"**

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How to avoid overfitting?

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Regularization in logistic regression

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Dropout in neural networks

Regularization

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$$\hat{\theta} = \arg\max_{\theta} \sum_{i=1}^{n} \log P(y^{(i)} \,|\, \mathbf{x}^{(i)}) - \alpha R(\theta)$$

Regularization

- A solution for overfitting: Add a regularization term $R(\theta)$ to the loss function
  - (for now written as maximizing logprob rather than minimizing loss)

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- Idea: choose an $R(\theta)$ that penalizes large weights
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  - fitting the data a little less well, with small weights

$$\hat{\theta} = \underset{\theta}{\arg\max} \sum_{i=1}^{n} \log P(y^{(i)} | \mathbf{x}^{(i)}) - \alpha R(\theta)$$

L2 / Ridge Regularization

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- The sum of the squares of the weights

$$R(\theta) = \|\theta\|_2^2 = \sum_{j=1}^{d} \theta_j^2$$

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L2 regularized objective function:

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L1 regularized objective function:

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