Graph-based Dependency Parsing

Chu-Liu-Edmonds and Camerini (k-best)

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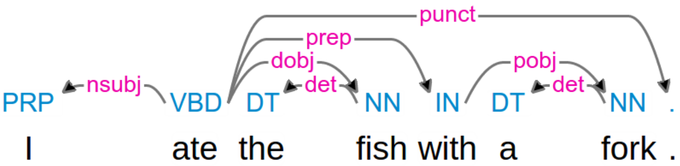
Dependency Parsing



TurboParser output from
`http://demo.ark.cs.cmu.edu/parse?sentence=I%20ate%20the%20fish%20with%20a%20fork.`

Dependency Parsing - Output Structure

A parse is an arborescence (aka directed rooted tree):

- Directed [Labeled] Graph
- Acyclic
- Single Root
- Connected and Spanning: $\exists$ directed path from root to every other word

Arc-Factored Model

Every possible labeled directed edge e between every pair of nodes gets a score, $\texttt{score}(e)$.

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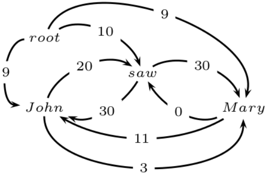
$G = \langle V, E \rangle =$



($O(n^2)$ edges)

Example from *Non-projective Dependency Parsing using Spanning Tree Algorithms McDonald et al., EMNLP '05*

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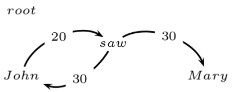
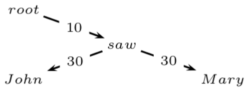
Best parse is:

$$A^{(1)} = \underset{A \subseteq G}{\arg\max} \sum_{e \in A} \texttt{score}(e)$$





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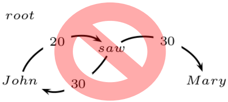

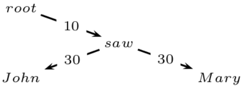

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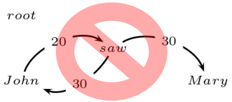

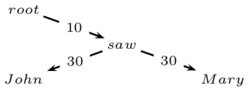

The Chu-Liu-Edmonds algorithm finds this argmax.

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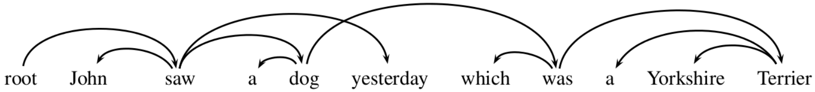
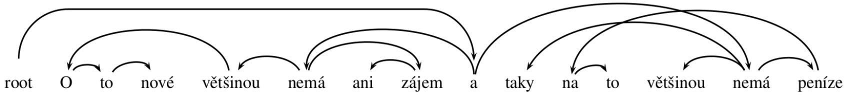
Projective / Non-projective

- Some parses are projective: edges don't cross
- Most English sentences are projective, but non-projectivity is common in other languages (e.g. Czech, Hindi)

Non-projective sentence in English:



and Czech:



He is mostly not even interested in the new things and in most cases, he has no money for it either.

Examples from *Non-projective Dependency Parsing using Spanning Tree Algorithms McDonald et al., EMNLP '05*

Dependency Parsing Approaches

- Chart (Eisner, CKY)
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  - *Only* produces projective parses
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- Shift-reduce
  - "Pseudo-projective" trick can capture some non-projectivity
  - $O(n)$ (*fast!*), but inexact
- Graph-based (MST)
  - Can produce projective *and* non-projective parses
  - $O(n^2)$ for arc-factored

Chu-Liu-Edmonds

Chu and Liu '65, On the Shortest Arborescence of a Directed Graph, Science Sinica

Edmonds '67, Optimum Branchings, JRNBS

Chu-Liu-Edmonds - Intuition

Every non-`ROOT` node needs exactly 1 incoming edge

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- C also needs an incoming edge.
- Choosing an incoming edge for C *determines* which edge to kick out

Chu-Liu-Edmonds

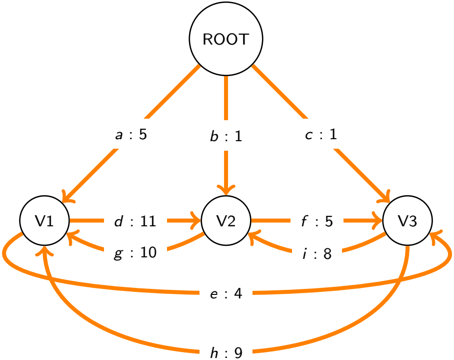
Consists of two stages:

- Contracting
- Expanding

Chu-Liu-Edmonds - Contracting Stage

- For each non-`ROOT` node v, set `bestInEdge`$[v]$ to be its highest scoring incoming edge.
- If a cycle C is ever formed:
  - contract the nodes in C into a new node v_C
  - edges incoming to any node in C now get destination v_C
  - edges outgoing from any node in C now get source v_C
  - For each node u in C, and for each edge e incoming to u from outside of C:
    - add `bestInEdge`[u] to `kicksOut`$[e]$, and
    - set the score of e to be `score`$[e]$ − `score`[`bestInEdge`$[u]$].
- Repeat until every non-`ROOT` node has an incoming edge and no cycles are formed

An Example - Contracting Stage



	bestInEdge
V1	
V2	
V3	

	kicksOut
a	
b	
c	
d	
e	
f	
g	
h	
i	

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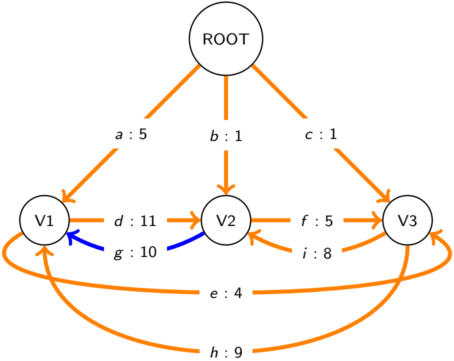

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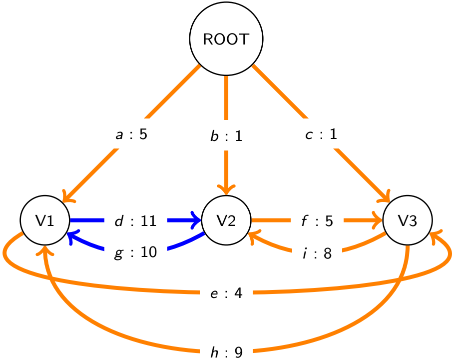

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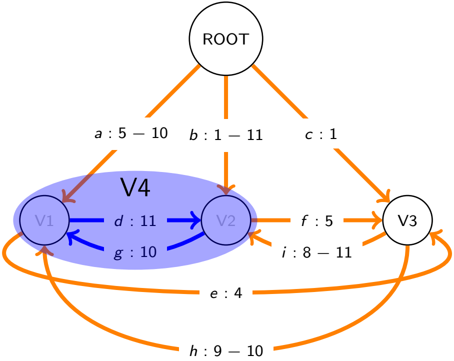

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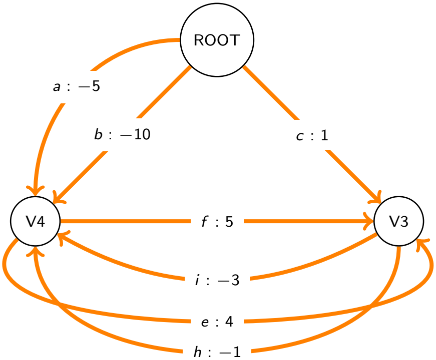

	bestInEdge
V1	g
V2	d
V3	
V4	

	kicksOut
a	g
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g	
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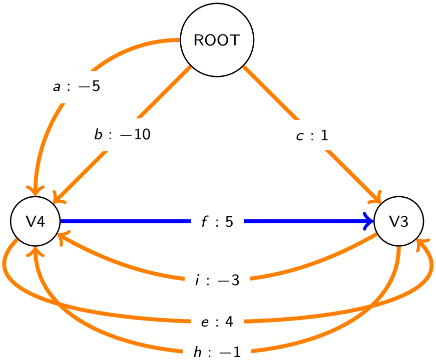

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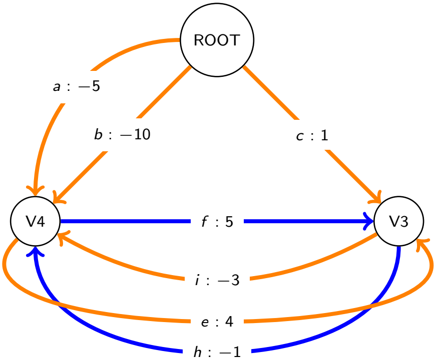

	bestInEdge
V1	g
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V3	f
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a	g
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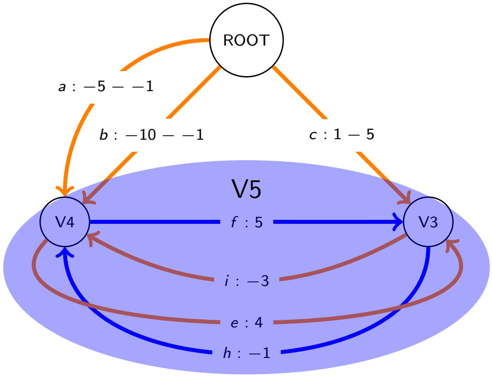

	bestInEdge
V1	g
V2	d
V3	f
V4	h
V5	

	kicksOut
a	g, h
b	d, h
c	f
d	
e	
f	
g	
h	g
i	d

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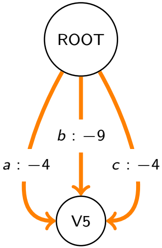

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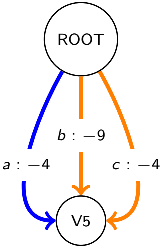

	bestInEdge
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a	g, h
b	d, h
c	f
d	
e	f
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i	d

Chu-Liu-Edmonds - Expanding Stage

After the contracting stage, every contracted node will have exactly one `bestInEdge`. This edge will kick out one edge inside the contracted node, breaking the cycle.

- Go through each `bestInEdge` e in the *reverse* order that we added them
- lock down e, and remove every edge in `kicksOut`(e) from `bestInEdge`.

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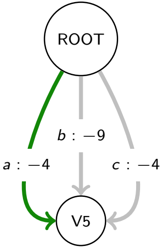

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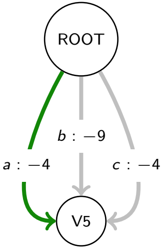

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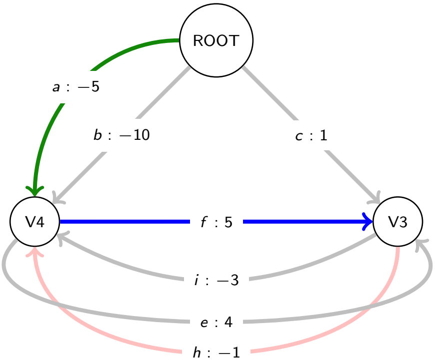

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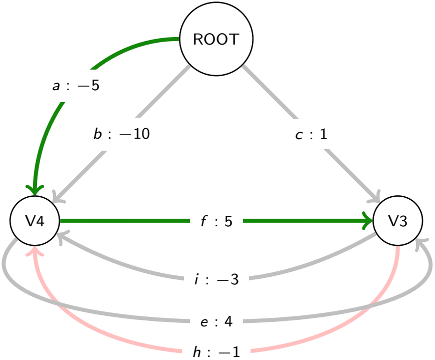

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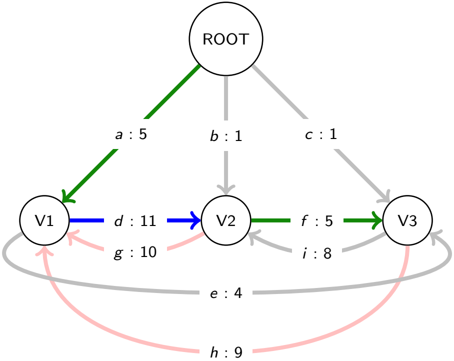

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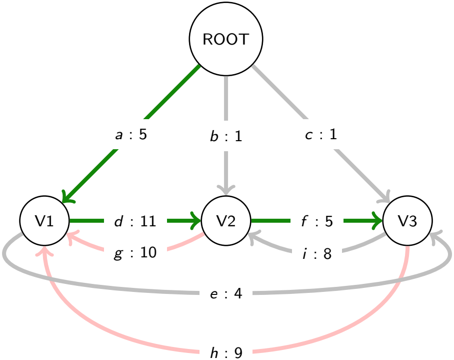

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Chu-Liu-Edmonds - Recursive Definition

```
def Get1Best(⟨V, E⟩, ROOT):
    """ returns best arborescence as a map from each node to its parent """
    for v in V \ ROOT:
        bestInEdge[v] ← arg max_{u∈V} score[(u, v)]

        if bestInEdge contains a cycle C:
            # build a new graph in which C is contracted into a single node
            v_C ← new Node
            V′ ← V ∪ {v_C} \ C
            E′ ← ∅
            for e = (t, u) in E:
                if t ∉ C and u ∉ C:
                    e′ ← e
                elif t ∈ C and u ∉ C:
                    e′ ← new Edge (v_C, u)
                    score[e′] ← score[e]
                elif u ∈ C and t ∉ C:
                    e′ ← new Edge (t, v_C)
                    kicksOut[e′] ← bestInEdge[u]
                    score[e′] ← score[e] − score[kicksOut[e′]]
                real[e′] ← e                                  # remember the original

                E′ ← E′ ∪ {e′}
            A ← Get1Best(⟨V′, E′⟩, ROOT)
            return {real[e′] | e′ ∈ A} ∪ (C_E \ {kicksOut[A[v_C]]})

    return bestInEdge
```

Chu-Liu-Edmonds - Notes

- Efficient implementation:
  Tarjan '77, Finding Optimum Branchings, Networks
  Not recursive. Uses a union-find (a.k.a. disjoint-set) data structure to keep track of collapsed nodes.

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- Even more efficient:
  Gabow et al. '86, Efficient Algorithms for Finding Minimum Spanning Trees in Undirected and Directed Graphs, Combinatorica
  Uses a Fibonacci heap to keep incoming edges sorted.
  Describes how to constrain `ROOT` to have only one outgoing edge

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  Uses a Fibonacci heap to keep incoming edges sorted.
  Describes how to constrain `ROOT` to have only one outgoing edge
- There is a version where you don't have to specify `ROOT`

Camerini

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Find *exact* k*-best* parses of a sentence given the weights of the graph

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Find *exact* k*-best* parses of a sentence given the weights of the graph

But why?

- Model might not be correct, rerank k-best parses
- Constrained models (think global features)

State of the art

- MSTParser and MaltParser produce an *approximate* k-best list
- TurboParser has no k-best feature

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5. Partition the whole search space into two smaller subspaces.

Partition the solution space

Let `reqd` = set of edges that must be included
and `banned` = set of edges that must be excluded.

Partitioning the solution space

$$\texttt{reqd} = \emptyset$$
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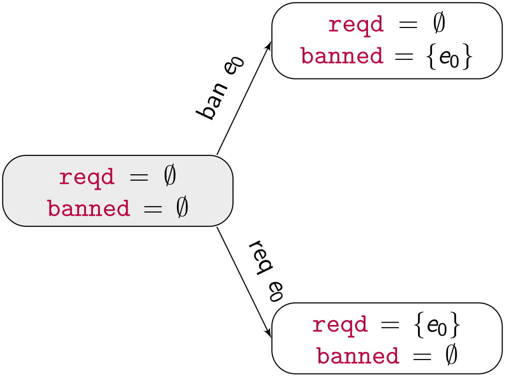

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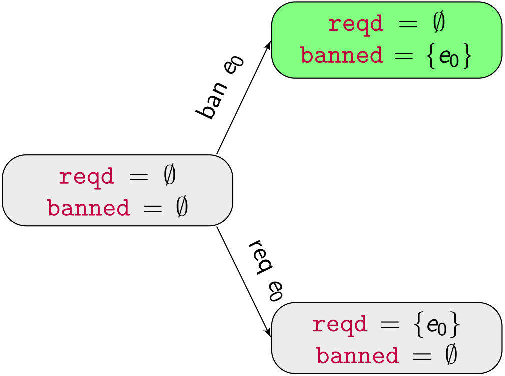

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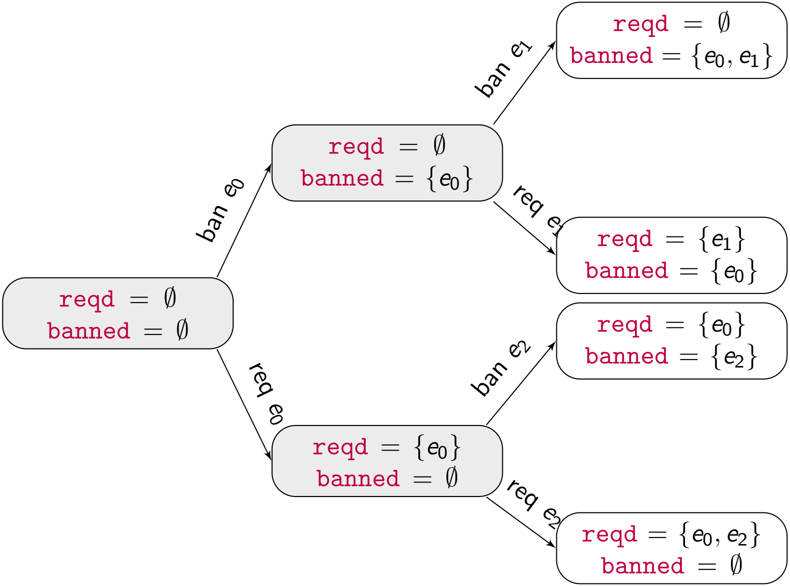
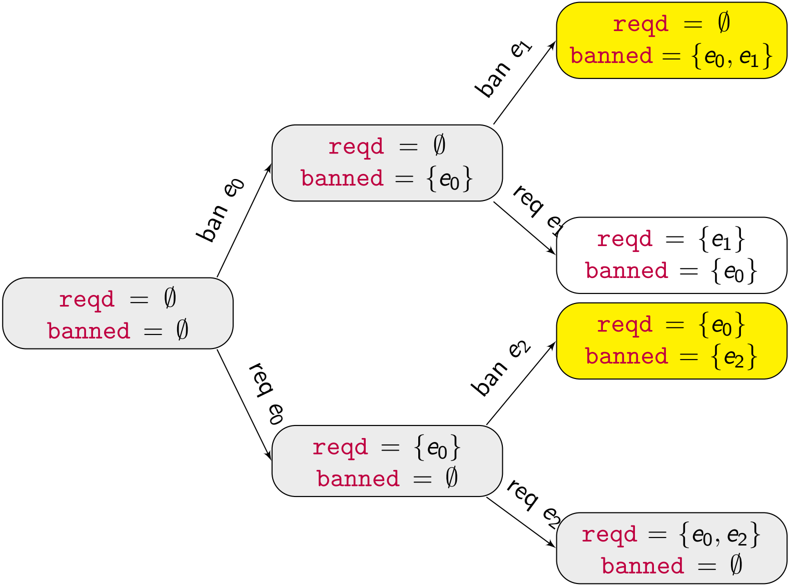
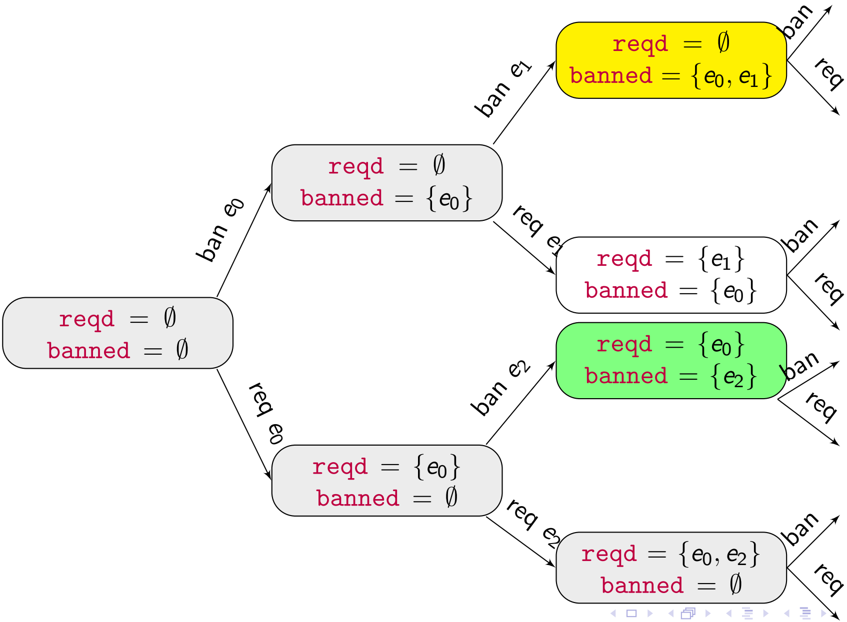


Partitioning the solution space



Partitioning the solution space



Outline of the rest of the talk

- Find best arborescence A s.t. $\texttt{reqd} \subseteq A \subseteq E \setminus \texttt{banned}$
  Algorithm $\texttt{GetConstrained1Best}(\texttt{G}, \texttt{ROOT}, \texttt{reqd}, \texttt{banned})$
- Find an edge $e \in A \setminus \texttt{reqd}$ that defines the next partition.
  Algorithm $\texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A, \texttt{reqd}, \texttt{banned})$
- Smart way to search the subspace of solutions
  Algorithm $\texttt{GetKBest}(\texttt{G}, \texttt{ROOT}, \texttt{k})$

Algorithm `GetConstrained1Best`(G, ROOT, reqd, banned)

Throw out edges before you feed the graph into `Get1Best`:

- Throw out every edge in `banned`
- Throw out every edge that *competes* with any edge in `reqd`
- Run `Get1Best`

Runtime
$O(n^2)$

Outline of the rest of the talk

- Find best arborescence A s.t. $\texttt{reqd} \subseteq A \subseteq E \setminus \texttt{banned}$
  ~~Algorithm GetConstrained1Best(G, ROOT, reqd, banned)~~
- Find an edge $e \in A \setminus \texttt{reqd}$ that defines the next partition.
  Algorithm FindEdgeToBan($\texttt{G}, \texttt{ROOT}, A, \texttt{reqd}, \texttt{banned}$)
- Smart way to search the subspace of solutions
  Algorithm GetKBest($\texttt{G}, \texttt{ROOT}, \texttt{k}$)

Algorithm FindEdgeToBan(G, ROOT, A, reqd, banned)

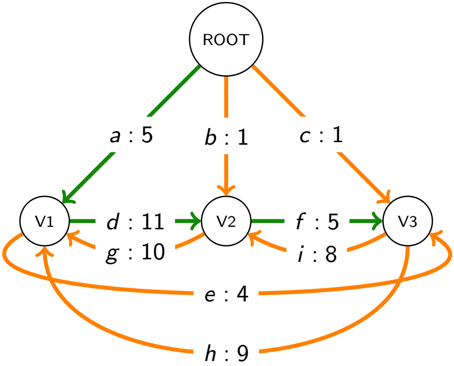
- Input (A, reqd, banned),
- For every edge e in $A \setminus$ reqd, find the next best alternative edge, alt(e)
  - this alternative cannot be in banned
  - the source of this alternative must not be lower down in the tree A
- Return eBan, the edge e in $A \setminus$ reqd with the highest scoring alternative
- Return diff = score(eBan) − alt(eBan)

Return variables eBan, diff

Runtime
$O(n^2)$

Example run FindEdgeToBan

$$\texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A^{(1)}, \texttt{reqd} = \emptyset, \texttt{banned} = \emptyset)$$



$$\texttt{diff} = +\infty, \ \texttt{eBan} = \emptyset$$

Example run FindEdgeToBan

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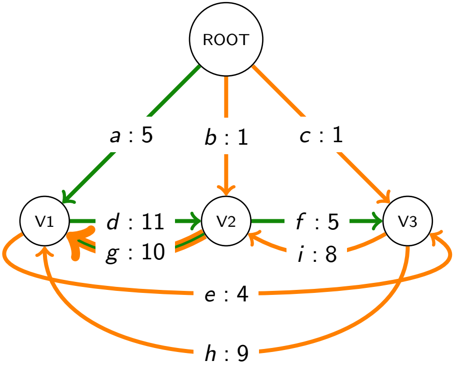

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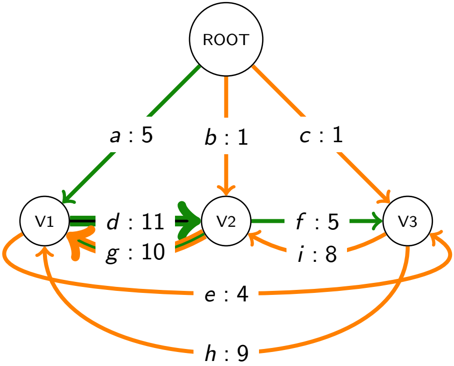

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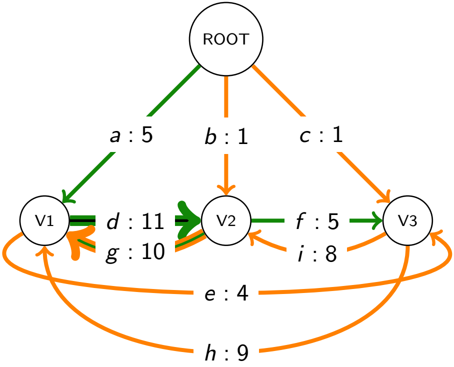
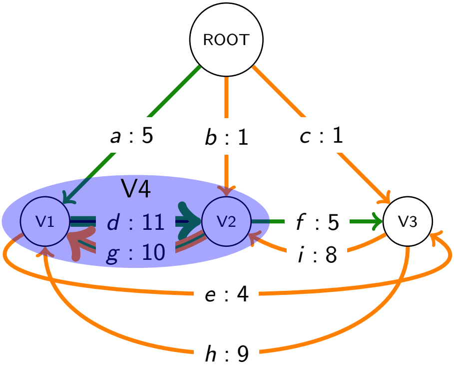


$$\texttt{alt}(d) = b$$

$$\texttt{diff} = 10, \ \texttt{eBan} = d$$

Example run FindEdgeToBan

$\texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A^{(1)}, \texttt{reqd} = \emptyset, \texttt{banned} = \emptyset)$



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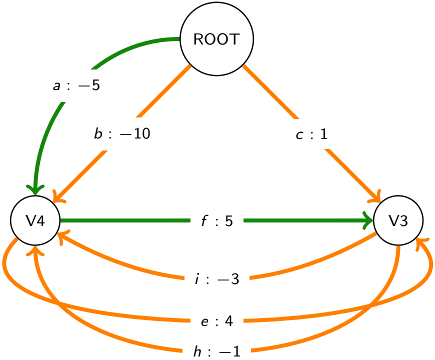

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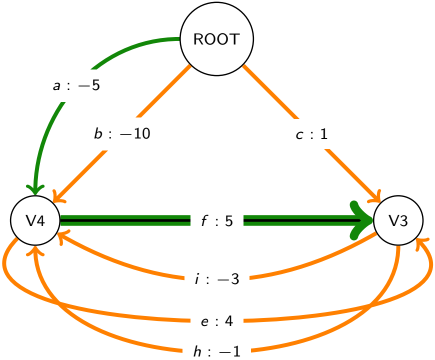

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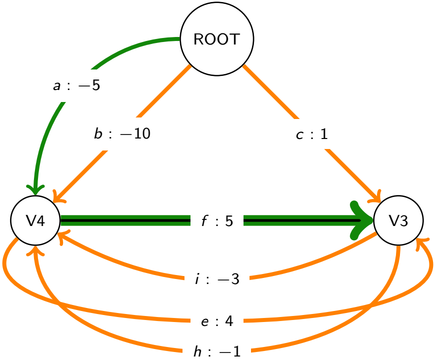
Example run `FindEdgeToBan`

$$\texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A^{(1)}, \texttt{reqd} = \emptyset, \texttt{banned} = \emptyset)$$

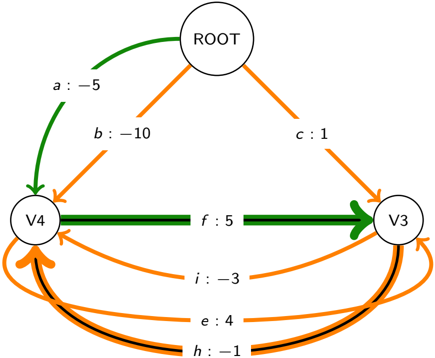


$$\texttt{alt}(f) = e$$

$$\texttt{diff} = 1, \; \texttt{eBan} = f$$

Example run FindEdgeToBan

$$\texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A^{(1)}, \texttt{reqd} = \emptyset, \texttt{banned} = \emptyset)$$



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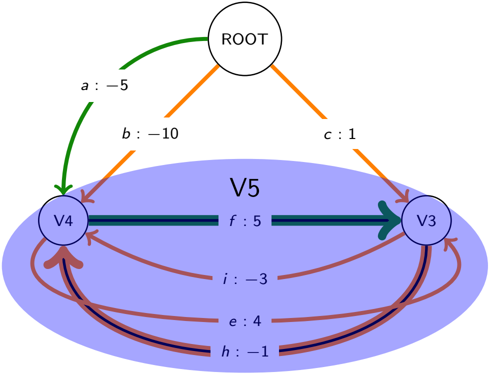

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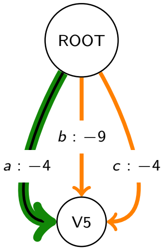

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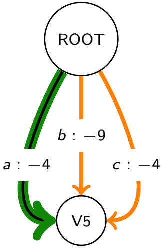
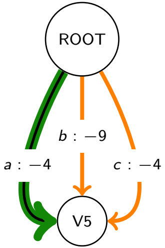


$$\texttt{alt}(a) = c$$

$$\texttt{diff} = 0, \ \texttt{eBan} = a$$

Example run FindEdgeToBan

$$\texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A^{(1)}, \texttt{reqd} = \emptyset, \texttt{banned} = \emptyset)$$



$$\texttt{alt}(a) = c$$

$$\texttt{diff} = 0, \ \texttt{eBan} = a$$

Outline of the rest of the talk

- Find best arborescence A s.t. $\texttt{reqd} \subseteq A \subseteq E \setminus \texttt{banned}$
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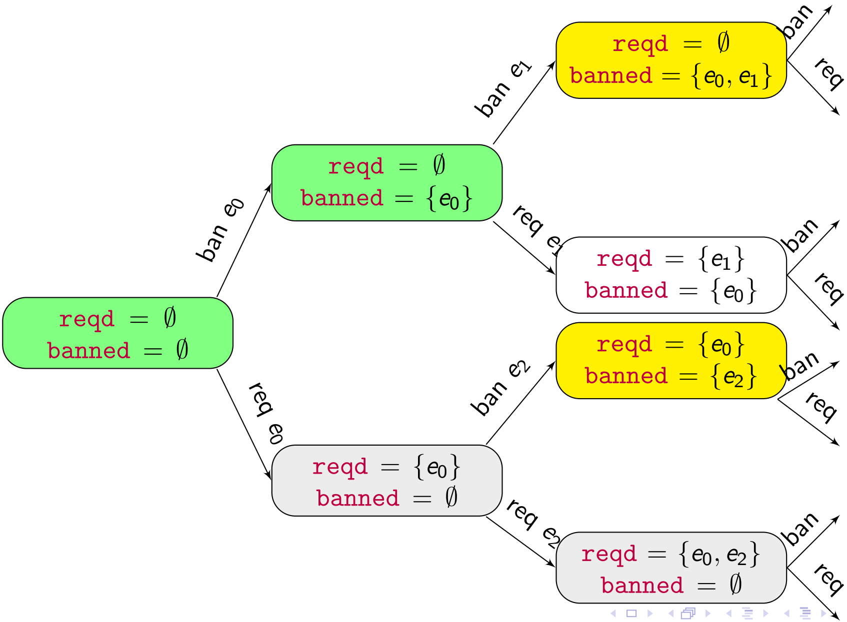
Revisit partitioning



Algorithm $\texttt{GetKBest}(\texttt{G}, \texttt{ROOT}, \texttt{k})$

- For every partition, save the following tuple: $(\texttt{wt}, \texttt{eBan}, A, \texttt{reqd}, \texttt{banned})$
- $A = \texttt{GetConstrained1Best}(\texttt{G}, \texttt{ROOT}, \texttt{reqd}, \texttt{banned})$ corresponds to the *best* solution in the partition
- $\texttt{diff}, \texttt{eBan} = \texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A, \texttt{reqd}, \texttt{banned})$
- $\texttt{wt} = \texttt{score}(A) - \texttt{diff}$
- Maintain a priority queue, $\texttt{Q}$ containing all tuples sorted by $\texttt{wt}$
- $\texttt{Q}$ determines which path to traverse in the search space

GetKBest

def GetKBest(G, $ROOT$, k):

  """ returns k-best arborescences """

  reqd $\leftarrow \emptyset$ banned $\leftarrow \emptyset$

  $A^{(1)} \leftarrow$ Get1Best($\langle G.V, G.E \rangle$, ROOT)

  diff, eBan $\leftarrow$ FindEdgeToBan(G, ROOT, $A^{(1)}$, reqd, banned)

  Q.*push*((score($A^{(1)}$) $-$ diff, eBan, $A^{(1)}$, reqd, banned))

  for j **in** $2 \ldots k$:

    (wt, eBan, $\bar{A}$, reqd, banned) $\leftarrow$ Q.*pop*()

    if $wt == -\infty$:

      return $A^{(1)}, \ldots, A^{(j-1)}$

    $\overline{\text{reqd}} \leftarrow$ reqd $\cup$ {eBan}

    $\overline{\text{banned}} \leftarrow$ banned $\cup$ {eBan}

    $A^{(j)} \leftarrow$ GetConstrained1Best(G, ROOT, reqd, banned′)

    diff, eBan $\leftarrow$ FindEdgeToBan(G, ROOT, $\bar{A}$, $\overline{\text{reqd}}$, banned)

    Q.*push*((score($\bar{A}$) $-$ diff, eBan, $\bar{A}$, $\overline{\text{reqd}}$, banned))

    diff, eBan $\leftarrow$ FindEdgeToBan(G, ROOT, $\bar{A}$, reqd, $\overline{\text{banned}}$)

    Q.*push*((wt $-$ diff, eBan, $\bar{A}$, reqd, $\overline{\text{banned}}$))

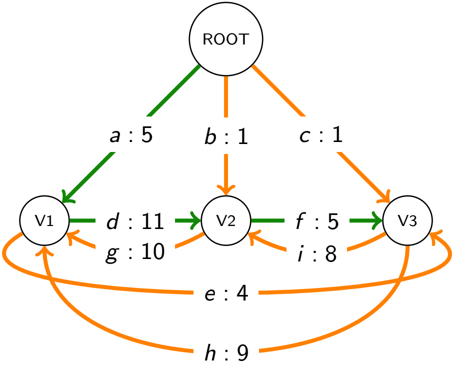
  return $A^{(1)}, \ldots, A^{(k)}$

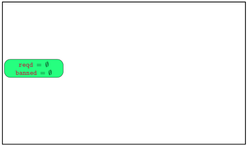
Runtime

$O(kn^2)$

GetKBest example : 1-best

$$A^{(1)} \leftarrow \texttt{GetConstrained1Best}(\texttt{G}, \texttt{ROOT}, \texttt{reqd} = \emptyset, \texttt{banned} = \emptyset)$$





Q

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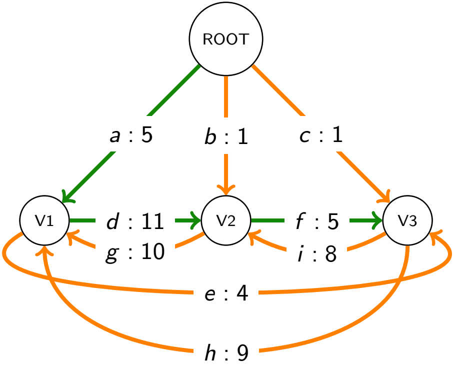
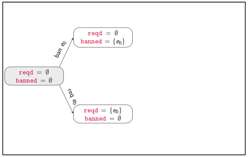


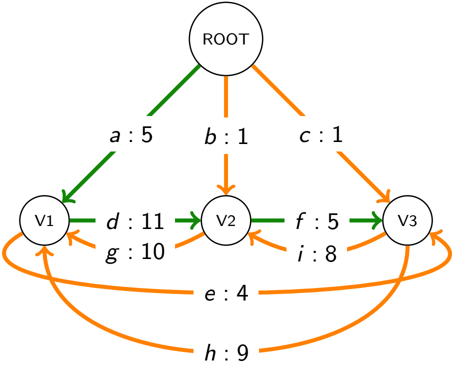


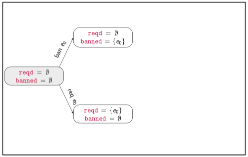
Q

$$(\texttt{diff} = 0, \texttt{eBan} = a) \leftarrow \texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A^{(1)}, \texttt{reqd} = \emptyset, \texttt{banned} = \emptyset)$$

GetKBest example : 1-best

$$A^{(1)} \leftarrow \texttt{GetConstrained1Best}(\texttt{G}, \texttt{ROOT}, \texttt{reqd} = \emptyset, \texttt{banned} = \emptyset)$$



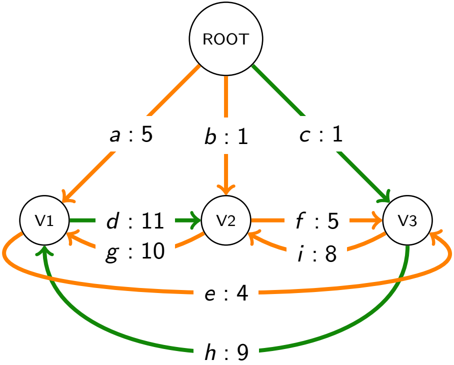


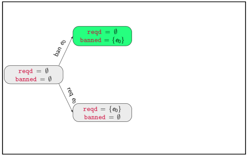
Q
$(21, a, A^{(1)}, \emptyset, \emptyset)$

$$(\texttt{diff} = 0, \texttt{eBan} = a) \leftarrow \texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A^{(1)}, \texttt{reqd} = \emptyset, \texttt{banned} = \emptyset)$$

GetKBest example: 2-best

$$A^{(2)} \leftarrow \texttt{GetConstrained1Best}(\texttt{G}, \texttt{ROOT}, \texttt{reqd} = \emptyset, \texttt{banned} = \{a\})$$





Q
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GetKBest example: 2-best

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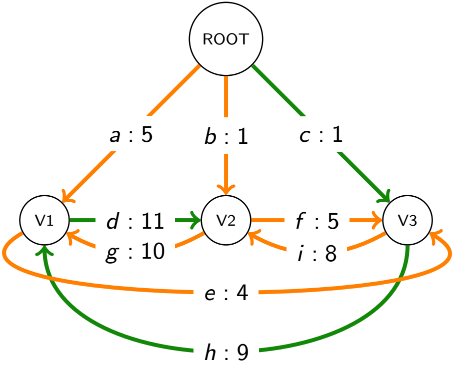
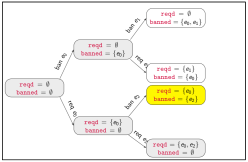


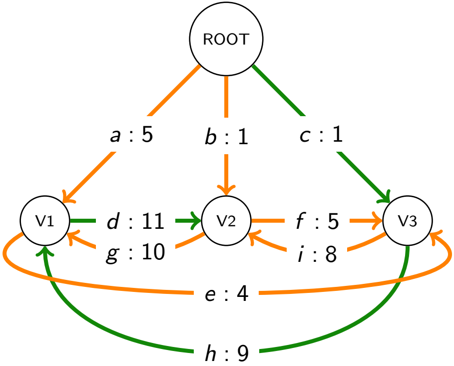


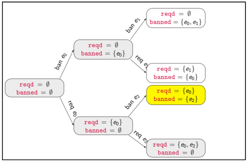
Q
$\cancel{(21, a, A^{(1)}, \emptyset, \emptyset)}$

$$(\texttt{diff} = 1,\ \texttt{eBan} = f) \leftarrow \texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A^{(1)}, \texttt{reqd} = \{a\}, \texttt{banned} = \emptyset)$$

GetKBest example: 2-best

$$A^{(2)} \leftarrow \texttt{GetConstrained1Best}(\texttt{G}, \texttt{ROOT}, \texttt{reqd} = \emptyset, \texttt{banned} = \{a\})$$



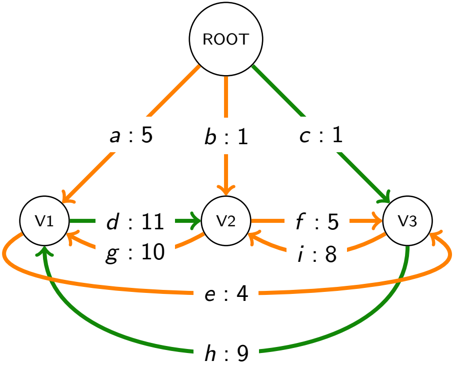


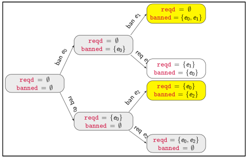
Q
$\cancel{(21, a, A^{(1)}, \emptyset, \emptyset)}$
$(20, f, A^{(1)}, \{a\}, \emptyset)$

$$(\texttt{diff} = 1,\ \texttt{eBan} = f) \leftarrow \texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A^{(1)}, \texttt{reqd} = \{a\}, \texttt{banned} = \emptyset)$$

$\texttt{GetKBest}$ example: 2-best

$$A^{(2)} \leftarrow \texttt{GetConstrained1Best}(\texttt{G}, \texttt{ROOT}, \texttt{reqd} = \emptyset, \texttt{banned} = \{a\})$$





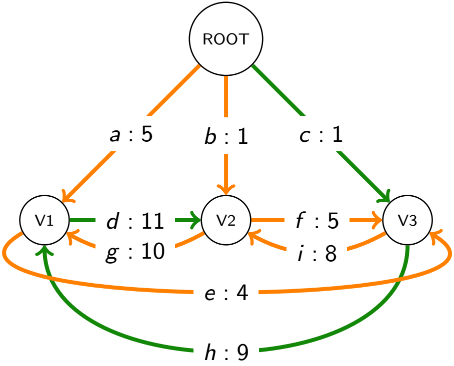
$\texttt{Q}$
$\cancel{(21, a, A^{(1)}, \emptyset, \emptyset)}$
$(20, f, A^{(1)}, \{a\}, \emptyset)$

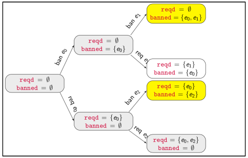
$$(\texttt{diff} = 1,\ \texttt{eBan} = f) \leftarrow \texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A^{(1)}, \texttt{reqd} = \{a\}, \texttt{banned} = \emptyset)$$

$$(\texttt{diff} = 2,\ \texttt{eBan} = h) \leftarrow \texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A^{(1)}, \texttt{reqd} = \emptyset, \texttt{banned} = \{a\})$$

GetKBest example: 2-best

$$A^{(2)} \leftarrow \texttt{GetConstrained1Best}(\texttt{G}, \texttt{ROOT}, \texttt{reqd} = \emptyset, \texttt{banned} = \{a\})$$





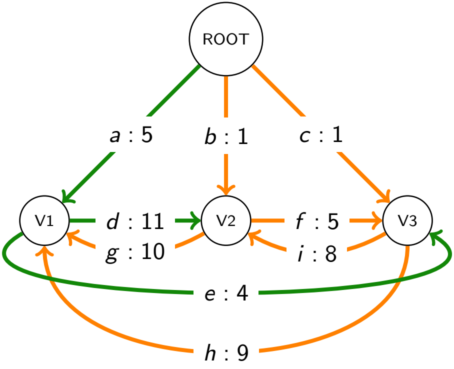
Q
$\cancel{(21, a, A^{(1)}, \emptyset, \emptyset)}$
$(20, f, A^{(1)}, \{a\}, \emptyset)$
$(19, h, A^{(2)}, \emptyset, \{a\})$

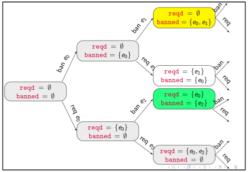
$$(\texttt{diff} = 1,\ \texttt{eBan} = f) \leftarrow \texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A^{(1)}, \texttt{reqd} = \{a\}, \texttt{banned} = \emptyset)$$

$$(\texttt{diff} = 2,\ \texttt{eBan} = h) \leftarrow \texttt{FindEdgeToBan}(\texttt{G}, \texttt{ROOT}, A^{(1)}, \texttt{reqd} = \emptyset, \texttt{banned} = \{a\})$$

GetKBest example : 3-best

$$A^{(3)} \leftarrow \texttt{GetConstrained1Best}(\texttt{G}, \texttt{ROOT}, \texttt{reqd} = \{a\}, \texttt{banned} = \{f\})$$





Q
~~$(21, a, A^{(1)}, \emptyset, \emptyset)$~~
~~$(20, f, A^{(1)}, \{a\}, \emptyset)$~~
$(19, h, A^{(2)}, \emptyset, \{a\})$

Conclusion

- Graph-based formulation for dependency parsing
- 1-best algorithm by Chu-Liu-Edmonds
- k-best algorithm by Camerini